

1. [14 points] No work or explanation is expected on this page.
- a. [4 points] If the graph of the function $y = k(w)$ has a vertical asymptote $w = 5$ and a horizontal asymptote $y = 3$, then what, if any, asymptotes does the graph of $y = 2k(4w) + 3$ have? (Write “None” in the blank if no asymptotes of that type can be found from the information provided.)

Solution: The graph of $y = 2k(4w) + 3$ is obtained from the graph of $y = k(w)$ by first compressing horizontally by a factor of $1/4$ then stretching vertically by a factor of 2, and finally shifting the graph up by 3 units. These transformations send the vertical asymptote $w = 5$ to $w = 5/4$ and the horizontal asymptote $y = 3$ to $y = 9$.

Vertical: $w = 5/4$ **Horizontal:** $y = 9$

- b. [2 points] If the function $f(q)$ has a zero $q = -2$, find a zero of the function $4f(3(q - 1))$.

Solution: The graph of $4f(3(q - 1))$ is obtained from the graph of $f(q)$ by first compressing horizontally by a factor of $1/3$ (which sends $q = -2$ to $q = -2/3$), then shifting right by 1 unit (taking $q = -2/3$ to $q = 1/3$) and finally stretching vertically by a factor of 4 (which does not change the horizontal intercepts). Hence, $q = 1/3$ is a zero of $4f(3(q - 1))$.

An alternate approach: $4f(3(q - 1)) = 0$ if $f(3(q - 1)) = 0$ so since $f(-2) = 0$, the transformed function has a zero when $3(q - 1) = -2$. Solving this equation, we find $q = 1/3$.

Answer: $q = 1/3$

- c. [2 points] Is the function $p(t) = \sin(t) + \pi$ even, odd, or neither? *Circle ONE answer.*

Solution: The graph of $y = \sin(t)$ is symmetric about the origin as sine is an odd function. However, the graph of $\sin(t) + \pi$, which is the graph of $\sin(t)$ shifted up π units, is not symmetric about the origin. It is also not symmetric about the vertical axis. Therefore the function $\sin(t) + \pi$ is neither even nor odd.

even

odd

neither even nor odd

- d. [2 points] Is there a number whose natural log is 8,675,309? *Circle ONE answer.*

Solution: The natural log of $e^{8,675,309}$ is 8,675,309. (The range of the natural logarithm function is all real numbers.)

yes

no

- e. [4 points] Suppose $\cos(\phi) = -0.6$ and $\pi \leq \phi \leq 2\pi$. Find $\sin(\phi)$ and $\tan(\phi)$.

Solution: By the Pythagorean Identity, $\sin^2(\phi) + \cos^2(\phi) = 1$, so $\sin^2(\phi) + (-0.6)^2 = 1$. Solving this equation, we find $\sin^2(\phi) = 0.64$ so $\sin(\phi) = \pm\sqrt{0.64} = \pm 0.8$. Since $\pi \leq \phi \leq 2\pi$, the sign of $\sin(\phi)$ is negative. Hence $\sin(\phi) = -0.8$.

By definition, we then have $\tan(\phi) = \frac{\sin(\phi)}{\cos(\phi)} = \frac{-0.8}{-0.6} = \frac{4}{3}$.

Answers: $\sin(\phi) =$ -0.8 $\tan(\phi) =$ $4/3$