2. [12 points] For each equation below, solve EXACTLY for the specified variable. Show your work carefully and write your final answer on the answer blank provided.

a. [4 points] If $5e^{2t+3} = 6$, solve for t.

Solution: Taking the natural log of both sides of the equation, we get $\ln\left(5e^{2t+3}\right) = \ln(6)$. Using the properties of ln, we find $\ln(5) + (2t + 3) = \ln(6)$. We can rearrange to get $2t = \ln(6) - \ln(5) - 3$, so that $t = \dfrac{\ln(6) - \ln(5) - 3}{2}$.

Answer: The exact value of t is $\dfrac{\ln(6) - \ln(5) - 3}{2}$ **or** $\dfrac{\ln(6/5) - 3}{2}$.

b. [4 points] If $\ln(3^x) = \ln(2^x) + 7$, solve for x.

Solution: Using the properties of the natural logarithm, this equation can be written $x\ln(3) = x\ln(2) + 7$. Collecting the x terms on one side of the equation, we find that $(\ln(3) - \ln(2))x = 7$, so that $x = \dfrac{7}{\ln(3) - \ln(2)}$.

Answer: The exact value of x is $\dfrac{7}{\ln(3) - \ln(2)}$ **or** $\dfrac{7}{\ln(3/2)}$.

c. [4 points] If $\log(w + 1) - \log(2 - w) = 2$, solve for w.

Solution: Using the properties of log, we can rewrite this equation as $\log\left(\frac{w+1}{2-w}\right) = 2$. By the definition of the logarithm (or raising 10 to the power of both sides), we get $\dfrac{w+1}{2-w} = 10^2 = 100$. Clearing the denominator, we find $w + 1 = 100(2 - w)$. Collecting all of the w terms on one side, we get $101w = 199$, so $w = \dfrac{199}{101}$.

Answer: The exact value of w is $\dfrac{199}{101}$.