

3. [10 points] Below, you are given a table with some data about two functions, $f(x)$ and $g(x)$:

x	0	1	2
$f(x)$	2	-4	3
$g(x)$	0	5	-1

In addition to this table, we are told the following facts:

- $f(x)$ is an even function.
- $g(x)$ is an odd function.
- There is a third function $h(x)$. The graph of $y = h(x)$ is obtained by shifting the graph of $y = f(x)$ one unit to the right.

Use this information to answer the questions below.

- a. [2 points] Compute $h(1)$.

Solution: Since the graph of $y = h(x)$ is obtained by shifting the graph of $y = f(x)$ one unit to the right, we have $h(x) = f(x - 1)$. So, $h(1) = f(1 - 1) = f(0) = 2$.

Alternatively, shifting the graph of $y = f(x)$ one unit to the right takes the point $(0, 2)$ to the point $(1, 2)$. So $(1, 2)$ is a point on the graph of $y = h(x)$ and thus $h(1) = 2$.

Answer: $h(1) =$ 2.

- b. [2 points] Compute $f(-1)$.

Solution: We are told $f(x)$ is even, so $f(x) = f(-x)$. Therefore, $f(-1) = f(1) = -4$.

Answer: $f(-1) =$ -4.

- c. [2 points] Compute $g(2) + g(-2)$.

Solution: We are told $g(x)$ is odd, so $g(-x) = -g(x)$. Therefore, $g(-2) = -g(2) = 1$, so $g(2) + g(-2) = -1 + 1 = 0$.

Answer: $g(2) + g(-2) =$ 0.

- d. [2 points] If $j(x) = f(2x) + 1$, compute the value of $j(1)$.

Solution: We have $j(1) = f(2 \cdot 1) + 1 = f(2) + 1 = 3 + 1 = 4$.

Answer: $j(1) =$ 4.

- e. [2 points] If $g(x) = k(-2x)$, compute the value of $k(-4)$.

Solution: The relationship between $g(x)$ and $k(x)$ shows us that the graph of $y = g(x)$ can be obtained from the graph of $y = k(x)$ by compressing horizontally by a factor of $\frac{1}{2}$ and reflecting across the vertical axis. From this, we can see that $g(2) = k(-4)$. From the table, we know $g(2) = -1$, so $k(-4) = -1$.

Alternatively, since $k(-2x) = g(x)$, we see that $k(-4) = k(-2 \cdot 2) = g(2) = -1$.

Answer: $k(-4) =$ -1.