4. [13 points] *For each problem on this page, show your work step-by-step. (Don't forget to use appropriate units in your answers.)*

Ozone is a molecule consisting of three oxygen atoms that is unstable and decays to the stable form of oxygen. The half-life of gaseous ozone at a temperature of 20°C is 3 days (72 hours).

a. [4 points] Find the continuous hourly percent decay rate of gaseous ozone at 20°C. *Give your answer in exact form.*

Solution: If the initial amount of gaseous ozone is a and the continuous hourly percent decay rate of gaseous ozone is k, then the amount of ozone after t hours is given by ae^{kt}. Since the half-life is given as 72 hours, then after 72 hours, the amount of ozone is $0.5a$, i.e. we have the equation $0.5a = ae^{72k}$. Then $0.5 = e^{72k}$, so, by the definition of ln (or taking the natural logarithm of both sides) we have $\ln(0.5) = 72k$ so $k = \dfrac{\ln(0.5)}{72}$.
Note: $\frac{\ln(0.5)}{72} \approx -0.009627$, so it decays at a continuous rate of about 0.9627% per hour.

Answer: $\dfrac{\ln(0.5)}{72}$

b. [4 points] At a temperature of 20°C, how long does it take for the amount of gaseous ozone to be reduced by 90%? *Give your answer in exact form.*

Solution: The amount of gaseous ozone remaining from an initial quantity a after t hours is ae^{kt} where $k = \ln(0.5)/72$ (as found in part (a)). When the amount has been reduced by 90%, the amount left is $0.1a$, so we have $0.1a = ae^{kt}$. Then $0.1 = e^{kt}$ so by the definition of ln (or taking the natural log of both sides), we have $\ln(0.1) = kt$ so $t = \ln(0.1)/k$. Using our value of k from part (a), this is $t = \dfrac{\ln(0.1)}{\ln(0.5)/72} = \dfrac{72\ln(0.1)}{\ln(0.5)}$.
Note: $\frac{72\ln(0.1)}{\ln(0.5)} \approx 239.1788$, so it takes approximately 239.2 hours for gaseous ozone to be reduced by 90%.

Answer: $\dfrac{72\ln(0.1)}{\ln(0.5)}$ **hours**

When ozone is dissolved in water, it is referred to as "aqueous ozone." At a temperature of 20°C, aqueous ozone decays at a rate of 12.5% per hour.

c. [5 points] Suppose that in a 20°C lab, the amount of aqueous ozone is initially 5 times the amount of gaseous ozone. When will the two amounts be equal? *Give your answer in exact form.*

Solution: Let g be the initial amount of gaseous ozone. Then the initial amount of aqueous ozone is $5g$, and, since its decay rate is 12.5% per hour, the amount of aqueous ozone remaining after t hours is $(5g)(0.875)^t$. From part (a) above, the amount of gaseous ozone remaining after t hours is $ge^{t\ln(0.5)/72}$. So, we are to find the value of t when these two amounts are equal, i.e. to solve the equation $ge^{t\ln(0.5)/72} = (5g)(0.875)^t$. Dividing both sides by g gives us $e^{t\ln(0.5)/72} = (5)(0.875)^t$. Taking the natural log of both sides of this equation, we then find $\ln\left(e^{t\ln(0.5)/72}\right) = \ln(5(0.875)^t)$ so $t\ln(0.5)/72 = \ln(5)+t\ln(0.875)$. Collecting like terms and factoring out t then gives us $t(\ln(0.5)/72 - \ln(0.875)) = \ln(5)$ so $t = \dfrac{\ln(5)}{\ln(0.5)/72 - \ln(0.875)}$. (Note that this is approximately 13 hours.)

Answer: $\dfrac{\ln(5)}{\ln(0.5)/72 - \ln(0.875)}$ **hours**