5. [13 points] In Western Japan (as in many other places around the world), electrical outlets supply power in the form of alternating current, which means that the voltage changes over time. The voltage goes from a maximum of 141 volts to a minimum of −141 volts and back again, 60 times every second. Let $V(t)$ be the voltage of an electrical outlet in Western Japan, where t is time, in seconds. Assume that $V(t)$ is obtained from the function $\sin(t)$ by performing shifts, stretches and/or reflections, and that at time $t = 0$, the voltage of the outlet is at 141 volts.

a. [5 points] Find the period, amplitude, and midline of $V(t)$. (Include units.)

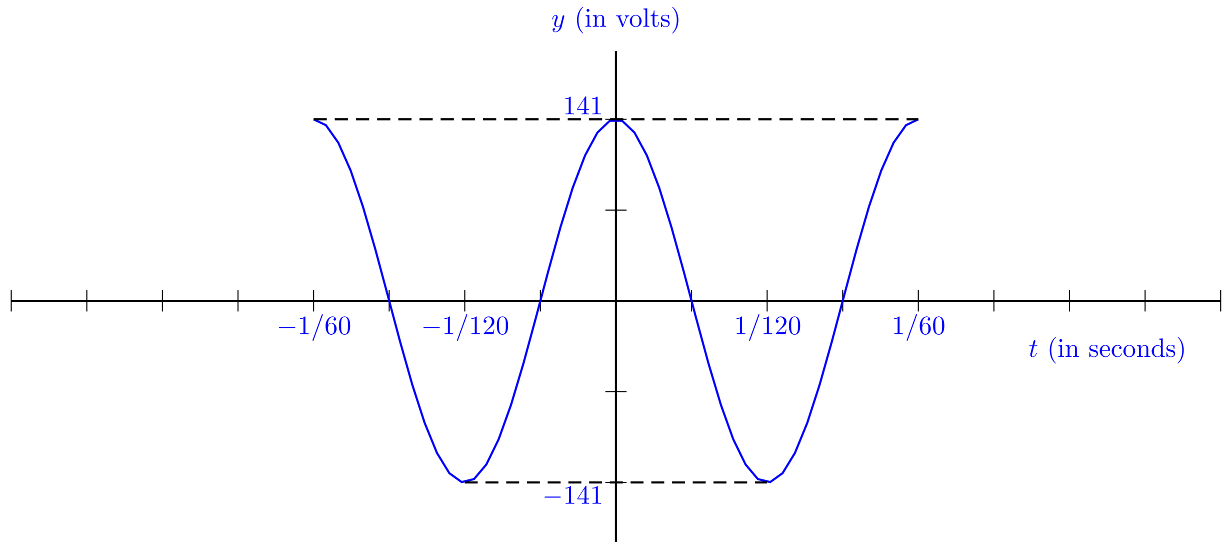
Solution:

Period: $1/60$ **of a second**

Amplitude: 141 **volts**

Midline: $y = 0$ **volts**

b. [5 points] On the axes provided, sketch a graph of $y = V(t)$ for two periods.
(*Clearly label the axes and be very careful with the shape and key features of your graph.*)



c. [3 points] The power, $P(t)$ (in Watts) dissipated by a particular electric lightbulb also varies with time. The graph of $y = P(t)$ is obtained from the graph of $y = V(t)$ by performing, in order, the following transformations:

1. A horizontal compression by a factor of $\frac{1}{2}$
2. A vertical compression by a factor of $\frac{60}{141}$
3. A shift upward by 60 units

Find a formula for $P(t)$ in terms of $V(t)$.

Solution: Horizontally compressing the graph of $y = V(t)$ by a factor of $1/2$ gives the graph of $y = V(2t)$. Vertically compressing this by a factor of $60/141$ gives the graph of $y = \dfrac{60}{141}V(2t)$, and finally shifting up by 60 units gives the graph of $y = \dfrac{60}{141}V(2t) + 60$

Answer: $P(t) = \dfrac{60}{141}V(2t) + 60$