8. [14 points] There is a bicycle wheel surrounded by a tire of uniform thickness. The wheel itself is 33 centimeters in radius, and the tire is 4 centimeters thick. The wheel has seven evenly-spaced spokes, one of which is initially pointing straight to the right. (See diagram below.)

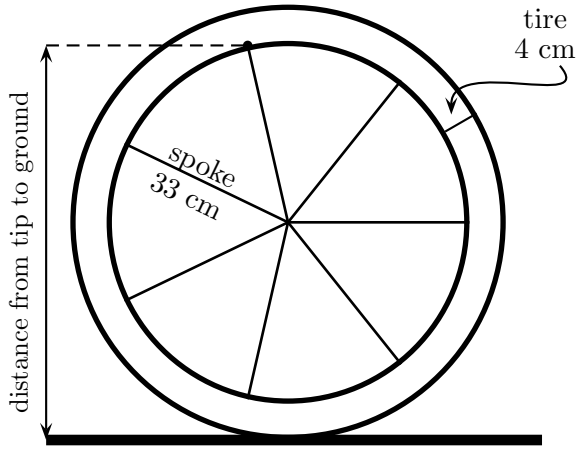


Figure not drawn to scale

a. [2 points] What is the exact angle (in radians) between two adjacent spokes?

Solution: Since there are 2π radians in a complete circle, the angle between two adjacent spokes is $\frac{2\pi}{7}$ radians.

Answer: $\frac{2\pi}{7}$ **radians**

b. [4 points] Find the distance from the tip of the highest spoke to the ground.
(*This distance is labeled as "distance from tip to ground" in the diagram above.*)

Solution: Since the angle between two adjacent spokes is $\frac{2\pi}{7}$, the highest spoke makes an angle of $2(\frac{2\pi}{7}) = \frac{4\pi}{7}$ with the rightward-pointing spoke. The radius of the wheel is 33 centimeters, so the tip of the highest spoke is $33\sin\left(\frac{4\pi}{7}\right)$ centimeters above the center of the wheel. The center of the wheel is 33 centimeters above the bottom of the wheel, which is 4 centimeters above the ground, so in total, the distance from the tip of the highest spoke to the ground is $33\sin\left(\frac{4\pi}{7}\right) + 33 + 4 \approx 69.1726$ centimeters.

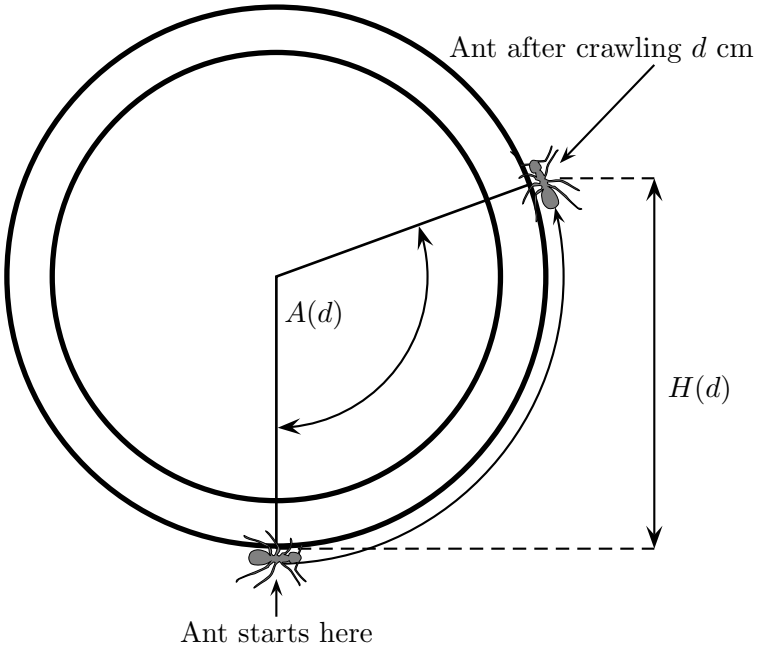
Answer: $33\sin\left(\frac{4\pi}{7}\right) + 37 \approx 69.1726$ **centimeters**

This problem continues on the next page.

This is a continuation of the problems from the previous page.

Recall: There is a bicycle wheel surrounded by a tire of uniform thickness. The wheel itself is 33 centimeters in radius, and the tire is 4 centimeters thick.

c. [4 points] One day, while the bicycle is parked, an ant crawls onto the bottom of the tire. The ant crawls for a distance of d centimeters along the outside of the tire. Let $A(d)$ denote the angle, measured in radians, through which the ant crawled. (See diagram on right.) Find a formula for $A(d)$ in terms of d.



Solution: The relationship between angle measurement A in radians and arc length d is $d = Ar$, where r is the radius of the circle. The ant is on the outside of the tire, so the radius of the circle of interest is $33 + 4 = 37$ centimeters. So, we get $d = A(d) \cdot 37$. We can solve for $A(d)$ to get $A(d) = \frac{d}{37}$.

Answer: $A(d) = \dfrac{d}{37}$

d. [4 points] The ant from part (c), after crawling through a distance of d centimeters, drops off of the tire and falls to the ground. Let $H(d)$ denote the distance, in centimeters, that the ant falls. (See diagram above.) Find a formula for $H(d)$ in terms of d.

Solution: The angle between the bottom of the wheel and the rightward-pointing horizontal is a quarter-circle, or $\frac{2\pi}{4} = \frac{\pi}{2}$ radians. The ant has crawled through an angle of $A(d)$ from the bottom of the wheel, so the ant's position makes an angle of $A(d) - \frac{\pi}{2} = \frac{d}{37} - \frac{\pi}{2}$ with the horizontal. At this point, the ant is a distance of $37 \sin\left(\frac{d}{37} - \frac{\pi}{2}\right)$ centimeters above the center of the wheel, so the distance the ant falls in total is $H(d) = 37 + 37 \sin\left(\frac{d}{37} - \frac{\pi}{2}\right)$ centimeters.

Answer: $H(d) = 37 + 37 \sin\left(\dfrac{d}{37} - \dfrac{\pi}{2}\right)$