10. [8 points] The management of a new pizza restaurant (called "New Pizza Restaurant" or "NPR") believes that the number of pizzas the restaurant will sell each month is a function of the amount of money it spends on advertising that month. Let $P(A)$ be the average number of pizzas the restaurant expects to sell in a month when it spends A dollars on advertising. Market research suggests that $$P(A) = 600 + 50\ln(A+1).$$

Throughout this problem, show your work step-by-step and give all answers in **exact form** *or rounded accurately to at least two decimal places. Include units.*

a. [1 point] How many pizzas does NPR expect to sell in a month if they spend no money on advertising?

Solution: If no money is spent on advertising, then NPR expects to sell $P(0) = 600 + 50\ln(0+1) = 600 + 50\ln(1) = 600$ pizzas in a month.

Answer: 600 **pizzas**.

b. [3 points] How much money does NPR plan to spend on advertising in a month if they want to sell 1200 pizzas that month?

Solution: We solve for A in the equation $P(A) = 1200$.

$$\begin{aligned} P(A) &= 1200 \\ 600 + 50\ln(A+1) &= 1200 \\ 50\ln(A+1) &= 1200 - 600 = 600 \\ \ln(A+1) &= 600/50 = 12 \\ A + 1 &= e^{12} \\ A &= e^{12} - 1 \approx 162,753.79 \end{aligned}$$

Hence NPR plans to spend about \$162, 754 on advertising.

Answer: **about** \$162, 754.

c. [4 points] A competitor, "OPR", expects to sell an average of $300+45\ln((A+1)^2)$ pizzas in a month when it spends A dollars on advertising. Suppose that in December, OPR and NPR will spend the same amount on advertising and will both expect to sell the same number of pizzas. How much will each restaurant spend on advertising in December?

Solution: We solve for A in the equation $300 + 45\ln((A+1)^2) = P(A)$.

$$\begin{aligned} 300 + 45\ln((A+1)^2) &= 600 + 50\ln(A+1) \\ 300 + 90\ln(A+1) &= 600 + 50\ln(A+1) \\ 40\ln(A+1) &= 300 \\ \ln(A+1) &= 7.5 \\ A + 1 &= e^{7.5} \\ A &= e^{7.5} - 1 \approx 1807.04 \end{aligned}$$

Hence each restaurant will spend about \$1807 on advertising in December.

Answer: **about** \$1807.