1. [8 points] For each of the statements below, circle "**True**" if the statement is *definitely* true. Otherwise, circle "**False**". You do not need to show any work for this problem.

a. [2 points] The function $g(x) = 3^x + \left(\frac{1}{3}\right)^x$ is an even function.

$\boxed{\text{True}}$ False

b. [2 points] The graph of $y = \ln(10x)$ can be obtained from the graph of $y = \ln(x)$ by a vertical shift.

$\boxed{\text{True}}$ False

c. [2 points] The line $y = 3$ is a horizontal asymptote of the function $f(x) = e^{10000x} + 3$.

$\boxed{\text{True}}$ False

d. [2 points] The function $h(x) = 5\cos(3x)$ is an odd function.

True $\boxed{\text{False}}$

2. [6 points] The graph of the function $g(x)$ contains the point $(-6, 4)$. For each of the functions below, find the coordinates of one point that must be on the graph of the function. Write the coordinates of the point in the form (x, y) on the provided answer blank.
You do not have to show work for this problem.

a. [2 points]
If $h(x) = 0.25g(-0.5x)$, then the graph of $h(x)$ must contain the point $(12, 1)$.

Solution: The graph of $h(x) = 0.25g(-0.5x)$ can be obtained from the graph of $g(x)$ as follows:
- First, compress vertically by a factor of 0.25, taking the point $(-6, 4)$ to the point $(-6, 1)$.
- Next, stretch horizontally by a factor of 2 $(= \frac{1}{0.5})$, taking the point $(-6, 1)$ to the point $(-12, 1))$.
- Finally, reflect across the vertical axis, taking the point $(-12, 1)$ to the point $(12, 1)$.

Check: $h(12) = 0.25g(-0.5(12)) = 0.25g(-6) = 0.25(4) = 1$, so $(12, 1)$ is indeed on the graph of $h(x)$.

b. [2 points]
If $n(x) = g(x+3)-4$, then the graph of $n(x)$ must contain the point $(-9, 0)$.

Solution: The graph of $n(x) = g(x + 3) - 4$ can be obtained from the graph of $g(x)$ as follows:
- Shift the graph *down* 4 units, taking the point $(-6, 4)$ to the point $(-6, 0)$.
- Shift the resulting graph 3 units *to the left*, taking the point $(-6, 0)$ to the point $(-9, 0)$.

Check: $n(-9) = g(-9 + 3) - 4 = g(-6) - 4 = 4 - 4 = 0$, so $(-9, 0)$ is indeed on the graph of $n(x)$.

c. [2 points]
If $p(x) = -3g(2x-4)$, then the graph of $p(x)$ must contain the point $(-1, -12)$.

Solution: By factoring, we rewrite $p(x) = -3g(2x - 4)$ as $p(x) = -3g(2(x - 2))$. The graph of $p(x) = -3g(2(x - 2))$ can be obtained from the graph of $g(x)$ as follows:
- First, stretch vertically by a factor of 3, taking the point $(-6, 4)$ to the point $(-6, 12)$.
- Next, reflect across the horizontal axis, taking the point $(-6, 12)$ to the point $(-6, -12)$.
- Then, compress horizontally by a factor of 1/2, taking $(-6, -12)$ to the point $(-3, -12)$.
- Finally, shift the resulting graph 2 units *to the right*, taking $(-3, -12)$ to the point $(-1, -12)$.

Check: $p(-1) = -3g(2(-1) - 4) = -3g(-6) = -3(4) = -12$, so $(-1, -12)$ is indeed on the graph of $p(x)$.