3. [5 points] A colony of bacteria triples in size every 6 days. What is the doubling time of this colony? (Show your work step-by-step, give your final answer in **exact form**, and *include units.*)

Solution: The colony is growing exponentially, so if its initial size is a, then its size after t days is ab^t for some constant b. Since the colony triples in size every 6 days, its population when $t = 6$ is $3a$, so $3a = ab^6$. Then $3 = b^6$ so $b = (3)^{1/6}$ and the colony size after t days is $a\left(3^{1/6}\right)^t = a(3^{t/6})$.

Let d be the doubling time of the colony. Then $2a = a(3^{d/6})$ so $2 = 3^{d/6}$.
Taking the natural logarithm of both sides of this equation and solving for d we find

$$\begin{aligned} 2a &= a(3^{d/6}) \\ 2 &= 3^{d/6} \\ \ln(2) &= \ln\left(3^{d/6}\right) \\ \ln(2) &= \frac{d}{6}\ln(3) \\ \frac{6\ln(2)}{\ln(3)} &= d \end{aligned}$$

Hence the doubling time of this colony is $\frac{6\ln(2)}{\ln(3)}$ days. (This is approximately 3.79 days.)

Answer: $\frac{6\ln(2)}{\ln(3)}$ **days**

4. [6 points] Let $G(m)$ be the mass (in grams) of the garbage in a dumpster m minutes before 8 am. For each of the functions below, find a formula by applying one or more appropriate transformations to the function G. (*In each case, your final answer should be a formula involving* G.)

a. [2 points] Let $K(m)$ be the mass (in **kilograms**) of the garbage in the dumpster m minutes before 8 am.

Answer: $K(m) =$ $0.001G(m)$.

b. [2 points] Let $L(h)$ be the mass (in kilograms) of the garbage in the dumpster h **hours** before 8 am.

Answer: $L(h) =$ $0.001G(60h)$.

c. [2 points] Let $T(h)$ be the mass (in kilograms) of the garbage in the dumpster h hours before **11 am**.

Solution: Note that $T(3) = L(0)$ (since in both cases this gives the mass in kg at 8 am). More generally, $T(h) = L(h-3) = 0.001G(60(h-3))$.

Answer: $T(h) =$ $0.001G(60(h-3))$ **or** $0.001G(60h-180)$.