5. [10 points] For potential partial credit, be sure to show your work.

a. [4 points] Suppose that the domain of \( f(t) \) is the interval \([-10, 20)\) and the range of \( f(t) \) is the interval \((-8, \infty)\). Find the domain and range of the function \( h(t) = 5f(-2t) + 6 \).

**Solution:** Note that “vertical transformations” affect only the range and “horizontal transformations” affect only the domain of the function. The graph of \( h(t) = 5f(-2t) + 6 \) can be obtained from the graph of \( f(t) \) as follows:

- Stretch vertically by a factor of 5 and then shift the resulting graph up 6 units. The range of the resulting function \((5f(t) + 6)\) is \((-34, \infty)\), and the domain is still \([-10, 20)\).
- Then reflect across the vertical axis and compress horizontally by a factor of \(1/2\) to get the graph of \( h(t) \). The resulting range is still \((-34, \infty)\) and the domain is \((-10, 5]\).

Domain: \((-10, 5]\)  
Range: \((-34, \infty)\)

b. [3 points] If a weight hanging on a string of length 6 feet swings through 11° on either side of the vertical, how long is the arc through which the weight moves from one high point to the next high point? (Give your answer in exact form and include units.)

**Solution:** Note that the total angle through which the weight swings is 22°. Using the formula for arclength, we find

\[
\text{arclength} = \text{radius} \cdot \text{angle (in radians)} \cdot \left(\frac{\pi}{180^\circ}\right) \\
= 6 \cdot \left(\frac{22^\circ \cdot \pi}{180^\circ}\right) \\
= \frac{22\pi}{30} = \frac{11\pi}{15}
\]

Answer: \(\frac{11\pi}{15}\) feet

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c. [3 points] The graph of \( T(x) \) can be obtained from the graph of \( \tan(x) \) by

- first stretching the graph horizontally (away from the vertical axis) by a factor of 3,
- then shifting the graph to the right 5 units,
- then reflecting the graph across the horizontal axis,
- and finally shifting the graph down 2 units.

Find a formula for \( T(x) \).

\[
\text{Answer: } T(x) = -\tan \left(\frac{1}{3}(x-5)\right) - 2
\]