5. [10 points] *For potential partial credit, be sure to show your work.*

a. [4 points] Suppose that the domain of $f(t)$ is the interval $[-10, 20)$ and the range of $f(t)$ is the interval $(-8, \infty)$. Find the domain and range of the function $h(t) = 5f(-2t) + 6$.

Solution: Note that "vertical transformations" affect only the range and "horizontal transformations" affect only the domain of the function. The graph of $h(t) = 5f(-2t)+6$ can be obtained from the graph of $f(t)$ as follows:
• Stretch vertically by a factor of 5 and then shift the resulting graph up 6 units. The range of the resulting function $(5f(t) + 6)$ is $(-34, \infty)$, and the domain is still $[-10, 20)$.
• Then reflect across the vertical axis and compress horizontally by a factor of 1/2 to get the graph of $h(t)$. The resulting range is still $(-34, \infty)$ and the domain is $(-10, 5]$.

Domain: $(-10, 5]$ **Range:** $(-34, \infty)$

b. [3 points] If a weight hanging on a string of length 6 feet swings through $11°$ on either side of the vertical, how long is the arc through which the weight moves from one high point to the next high point? (*Give your answer in exact form and include units.*)

Solution: Note that the total angle through which the weight swings is $22°$. Using the formula for arclength, we find

$$\begin{aligned} \text{arclength} &= \text{radius} \cdot \text{angle (in radians)} \\ &= 6 \cdot \left(22° \cdot \frac{\pi}{180°}\right) \\ &= \frac{22\pi}{30} = \frac{11\pi}{15} \end{aligned}$$

Answer: $\frac{11\pi}{15}$ **feet**

c. [3 points] The graph of $T(x)$ can be obtained from the graph of $\tan(x)$ by

- first stretching the graph horizontally (away from the vertical axis) by a factor of 3,
- then shifting the graph to the right 5 units,
- then reflecting the graph across the horizontal axis,
- and finally shifting the graph down 2 units.

Find a formula for $T(x)$.

Answer: $T(x) = -\tan\left(\frac{1}{3}(x-5)\right) - 2$