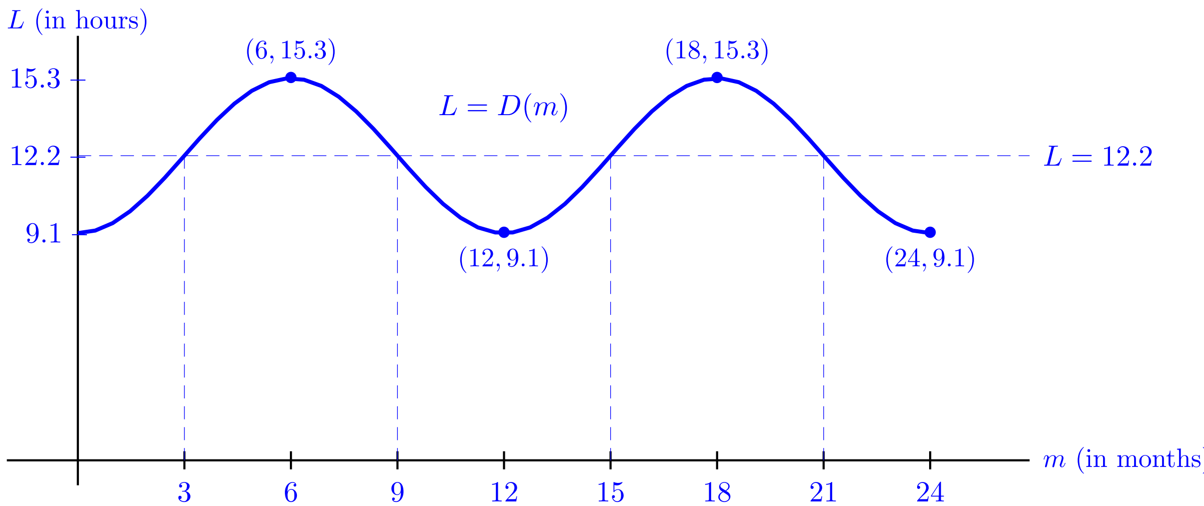
6. [14 points] The number of hours of daylight in Ann Arbor varies from a minimum of 9.1 hours of daylight on December 21 to a maximum of 15.3 hours of daylight on June 21 (and then back down to 9.1 hours on the following December 21). Let $L = D(m)$ be the number of hours of daylight in Ann Arbor on a day that is m months after December 21, 2010. Assume that $D(m)$ is a sinusoidal function.

a. [4 points] On the axes provided below, graph *two periods* of the function $L = D(m)$ starting with $m = 0$. (*Clearly label the axes and important points on your graph. Be very careful with the* **shape** *and* **key features** *of your graph.*)



b. [4 points] Find the period, amplitude, and midline of $L = D(m)$.
(*Include units* for the period and amplitude.)

Period: 12 **months**

Amplitude: 3.1 **hours**

Midline: $L = 12.2$

c. [4 points] Find a formula for $D(m)$.

Solution: Note that the graph begins at a minimum at $m = 6$ and then is increasing when it crosses the midline at $m = 3$. Using the data determined in part (b) above, two possible formulas are thus given by

$$D(m) = -3.1\cos\left(\frac{\pi}{6}m\right) + 12.2 \quad \text{and} \quad D(m) = 3.1\sin\left(\frac{\pi}{6}(m-3)\right) + 12.2.$$

Answer: $D(m) = -3.1\cos\left(\frac{\pi}{6}m\right) + 12.2$

d. [2 points] Use your formula from part (c) to estimate the number of hours of daylight in Ann Arbor on April 21. (Show your work and round your answer to the nearest 0.1 hour.)

Solution: April 21 is four months after December 21, so we need to compute $D(4)$.

$$D(4) = -3.1\cos\left(\frac{\pi}{6}(4)\right) + 12.2 = -3.1\cos\left(\frac{2\pi}{3}\right) + 12.2 = -3.1\left(-\frac{1}{2}\right) + 12.2 = 13.75.$$

Thus according to this model, there should be about 13.8 hours of daylight on April 21.

Answer: **About** 13.8 **hours of daylight**