7. [12 points] Solve each of the equations below. *Show your work step-by-step and write the solutions in* **exact form** *in the answer blanks provided.*

a. [3 points] $5(1.7)^{2y} = 2.4$

Solution: We first divide both sides of the equation by 5 and then use logarithms to find y.

$$\begin{aligned} 5(1.7)^{2y} &= 2.4 \\ (1.7)^{2y} &= \frac{2.4}{5} = 0.48 \\ \ln(1.7^{2y}) &= \ln(0.48) \\ 2y\ln(1.7) &= \ln(0.48) \\ y &= \frac{\ln(0.48)}{2\ln(1.7)} \end{aligned}$$

Answer: $y =$ $\dfrac{\ln(0.48)}{2\ln(1.7)}$

b. [3 points] $3t - 1 = \log\left(2(10)^{4.6t}\right)$

Solution: Using properties of logarithms, we see that $\log\left(2(10)^{4.6t}\right) = \log(2) + \log(10^{4.6t}) = \log(2) + 4.6t$, so it remains to solve the equation $3t - 1 = \log(2) + 4.6t$. Then we find $-1.6t = \log(2) + 1$ so $t = \dfrac{\log(2)+1}{-1.6}$

Answer: $t =$ $\dfrac{\log(2)+1}{-1.6}$

c. [3 points] $e^{\ln(w-4)} = \ln(3.2) - \ln(4)$

Solution: Applying basic properties of the natural logarithm, we see that $e^{\ln(w-4)} = w - 4$ and $\ln(3.2) - \ln(4) = \ln(\frac{3.2}{4}) = \ln(0.8)$. Thus $w - 4 = \ln(0.8)$ so $w = 4 + \ln(0.8)$. However, note that we cannot plug this value of w into the original equation (since this would involve $\ln(\ln(0.8))$, which is undefined because $\ln(0.8) < 0$). So, if there were a solution, to the equation, it would be $w = 4 + \ln(0.8)$, but there is actually no solution.

Answer: $w =$ **No solution**

d. [3 points] $\log(2p+1) - \log(p-3) = 3$

Solution: We apply a basic property of logarithms and then use the definition of the logarithm (or exponentiate).

$$\begin{aligned} \log(2p+1) - \log(p-3) &= 3 \\ \log\left(\frac{2p+1}{p-3}\right) &= 3 \\ \frac{2p+1}{p-3} &= 10^3 = 1000 \\ 2p+1 &= 1000(p-3) = 1000p - 3000 \\ 3001 &= 998p \\ \frac{3001}{998} &= p \end{aligned}$$

Answer: $p =$ $\dfrac{3001}{998}$