9. [11 points] *For this problem, show your work step-by-step and give all answers in* **exact form** *or accurate to at least three decimal places. Include units.*

The concentration (in milligrams per milliliter) of a certain experimental medication ("Medication E") in a patient's bloodstream t hours after injection is $C(t) = De^{-1.5t}$, where D is the concentration immediately after the injection.

a. [2 points] By what percent does the concentration of Medication E in the bloodstream decrease each hour after injection?

Solution: The hourly decay factor is $e^{-1.5} \approx 0.22313$, so the concentration of Medication E decreases by a factor of $1 - e^{-1.5} \approx 0.77687$ or about 77.687% per hour.

Answer: **About** 77.687%

b. [3 points] What is the half-life of Medication E in the bloodstream?

Solution: Suppose h is the half-life of Medication E in the bloodstream. Then $C(h) = \frac{1}{2}D$ so $De^{-1.5h} = \frac{1}{2}D$ and $e^{-1.5h} = \frac{1}{2}$. Hence $-1.5h = \ln(1/2)$ so $h = \frac{\ln(1/2)}{-1.5} \approx 0.462$. Therefore, the half-life is approximately 0.462 hours.

Answer: $\dfrac{\ln(1/2)}{-1.5} \approx 0.462$ **hours**

Suppose that a patient is given two injections (Medications A and B) at the same time.

- Medication A has an initial blood concentration of 3 mg/ml, and its concentration decreases at a *continuous* hourly rate of 25%.
- Medication B has an initial blood concentration of 4.5 mg/ml, and its concentration decreases at a *continuous* hourly rate of 30%.

Let $A(t)$ and $B(t)$ be the blood concentration (in mg/ml) of Medication A and of Medication B, respectively, t hours after the patient receives these injections.

c. [2 points] Find a formula for $A(t)$ and a formula for $B(t)$.

$A(t) =$ $3e^{-0.25t}$ $\qquad$ $B(t) =$ $4.5e^{-0.3t}$

d. [4 points] How long after the injections will the concentration of Medication B be only 2% more than the concentration of Medication A in the bloodstream?

Solution: The concentration of Medication B is 2% more than the concentration of Medication A when $B(t) = 1.02A(t)$, so we solve for t in this equation.

$$\begin{aligned}
B(t) &= 1.02A(t)\\
4.5e^{-0.3t} &= 1.02(3e^{-0.25t}) = 3.06e^{-0.25t}\\
\ln(4.5e^{-0.3t}) &= \ln(3.06e^{-0.25t})\\
\ln(4.5) + \ln(e^{-0.3t}) &= \ln(3.06) + \ln(e^{-0.25t})\\
\ln(4.5) - 0.3t &= \ln(3.06) - 0.25t\\
\ln(4.5) - \ln(3.06) = 0.05t \quad \text{so} \quad t &= \frac{\ln(4.5) - \ln(3.6)}{0.05} = \frac{\ln(4.5/3.06)}{0.05} \approx 7.713.
\end{aligned}$$

Hence the concentration of Medication B is 2% more than the concentration of Medication A $\dfrac{\ln(4.5/3.06)}{0.05}$ or about 7.713 hours after the injections.

Answer: $\dfrac{\ln(4.5/3.06)}{0.05} \approx 7.713$ **hours**