10. [10 points] Larry the llama and his family of five love having family game night! They find that the more soda they consume on game night, the more board games they can play. Let $b(z)$ be the number of board games they play when the family consumes z ounces of soda. After a few months of family game night, the family finds that

$$b(z) = 3 + 12\log\left(\frac{z}{p}\right)$$

where p is a positive constant.

a. [3 points] If Larry's family wants to play 13.5 board games, how many ounces of soda should they consume? (Your answer may involve p, but numbers should be in *exact form.*) *Show your work carefully.*

Solution: Since Larry's family wants to play 13.5 board games, we solve for z in the equation $b(z) = 13.5$. We have $3 + 12\log\left(\frac{z}{p}\right) = 13.5$ so $12\log\left(\frac{z}{p}\right) = 10.5$. Since $10.5/12 = 0.875$ we therefore have $\log\left(\frac{z}{p}\right) = 0.875$. By the definition of the logarithm (or "exponentiating"), we thus find $\frac{z}{p} = 10^{0.875}$, so $z = 10^{0.875}p$.

Answer: $10^{0.875}p$ **ounces of soda**

b. [4 points] *Note: This problem does not depend on part (a) above.*
Suppose the family normally drinks M ounces of soda on game night. How many more board games than usual do they play if they drink 5 times more soda than normal?
Show your work carefully. Your final answer should be a number, i.e. should not include any constants like p or M. Please round to the nearest 0.1 game.

Solution: If they drink 5 times more soda than normal, they drink $5M$ ounces of soda. If they drink $5M$ ounces of soda, they play $b(5M) = 3 + 12\log(5M/p)$ board games. Thus, they play $b(5M) - b(M)$ more board games than usual. We use basic properties of logarithms to simplify this expression.

$$\begin{aligned} b(5M) - b(M) &= \left(3 + 12\log\left(\frac{5M}{p}\right)\right) - \left(3 + 12\log\left(\frac{M}{p}\right)\right) \\ &= 12\log\left(\frac{5M}{p}\right) - 12\log\left(\frac{M}{p}\right) = 12\left(\log\left(\frac{5M}{p}\right) - \log\left(\frac{M}{p}\right)\right) \\ &= 12\log\left(\frac{5M/p}{M/p}\right) = 12\log(5) \approx 8.4. \end{aligned}$$

Answer: 8.4 **board games**

c. [3 points] *Note: This problem does not depend on parts (a) or (b) above.*
Suppose that Larry finds that $b(64) = 5$. Use this to solve *exactly* for p.
Show your work carefully.

Solution: If $b(64) = 5$, then we know that $3 + 12\log\left(\frac{64}{p}\right) = 5$ so $12\log\left(\frac{64}{p}\right) = 2$. Then $\log\left(\frac{64}{p}\right) = \frac{1}{6}$, so $\frac{64}{p} = 10^{\frac{1}{6}}$. Solving for p we find that $p = 64(10^{-\frac{1}{6}})$.

Answer: $p =$ $64(10^{-\frac{1}{6}})$