3. [12 points] Cleaver the beaver is building a large dam to protect against predators. After 4 hours of working, the dam he is building is 24 cm high. After 16 hours of working, the dam he is building is 180 cm high. Let $C(t)$ be the height of Cleaver's beaver dam, in cm, after he has been working for t hours. Assume that $C(t)$ is exponential.

a. [4 points] Find a formula for $C(t)$. *You must find your answer algebraically. All numbers in your formula should be in exact form.*

Solution: Since $C(t)$ is exponential, there are constants a and b so that $C(t) = ab^t$. We also have that $C(4) = 24$ and $C(16) = 180$. So, we have $24 = ab^4$ and $180 = ab^{16}$. Taking ratios, we find $\frac{180}{24} = \frac{ab^{16}}{ab^4}$ so $b^{12} = 7.5$ and $b = (7.5)^{1/12}$. We use the equation $ab^4 = 24$ to find the value for a and see that $a\left((7.5)^{1/12}\right)^4 = 24$ so $a = 24(7.5)^{-1/3}$. Thus, $C(t) = 24(7.5)^{-1/3}(7.5)^{t/12} = 24(7.5)^{(t-4)/12}$.

Answer: $C(t) = 24(7.5)^{-1/3}(7.5)^{t/12} = 24(7.5)^{(t-4)/12}$

b. [1 point] Find the continuous hourly growth rate of the height of Cleaver's dam. *Round your answer to the nearest* 0.01%.

Solution: To find the continuous hourly growth rate, we solve the equation $e^k = b$ for k. We have $e^k = (7.5)^{1/12}$ so $k = \frac{1}{12}\ln(7.5) = \frac{\ln 7.5}{12} \approx 16.79\%$.

Answer: 16.79%

Cleaver's neighbors, Anne and Barry, are also each building a dam, and they start working at the same time. Let $A(t)$ be the height, in cm, of Anne's dam t hours after she starts working on it, and let $B(t)$ be the height, in cm, of Barry's dam t hours after he starts working on it.

c. [2 points] Write an equation that expresses the following sentence:

"After they have been working for h hours, Anne's dam is 35% taller than Barry's dam."

Note: Your equation may involve A, B, and h.

Solution: If Anne's dam is 35% is taller than Barry's dam after working for h hours, then $A(h) = 1.35B(h)$.

Answer: $A(h) = 1.35B(h)$

- Anne's dam starts off 5 cm high, and she builds at a continuous hourly rate of 22%.
- Barry's dam starts off 12 cm high, and he builds at a constant rate of 4 cm per hour.

d. [2 points] Use the information above to find formulas for $A(t)$ and $B(t)$.

Solution: If Anne builds at a continuous hourly rate of 22% and her dam starts at 5 cm high, then $A(t) = 5e^{0.22t}$. If Barry builds at a constant rate of 4 cm per hour and his dam starts at 12 cm high, then $B(t) = 12 + 4t$.

Answers: $A(t) = 5e^{0.22t}$ and $B(t) = 12 + 4t$

e. [3 points] When will Anne's dam be 35% taller than Barry's dam? *Round your answer to the nearest* 0.01 *hour. Clearly indicate how you found your solution. (Remember item 7 from the instructions on the front page.)*

Solution: We want to solve $A(t) = 1.35B(t)$. We graph the functions $A(t)$ and $1.35B(t)$, and using the intersection feature of the calculator, we find that $t \approx -2.46$ or 12.93. Because Anne and Barry started building at time $t = 0$, only the positive solution makes sense in the context of this problem. Therefore, Anne's dam is 35% taller than Barry's dam approximately 12.93 hours after they started to build their dams.

Answer: **About 12.93 hours after starting to build**