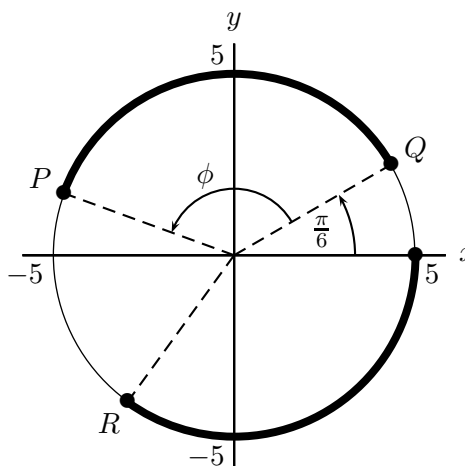


5. [13 points]

The problems on this page refer to the diagram to the right. As shown in the diagram, note the following:

- The points P , Q , and R are on the circle.
- The angle between the positive x -axis and the line segment from the origin to Q is $\frac{\pi}{6}$ radians.
- The angle between the line segment from the origin to Q and the line segment from the origin to P is ϕ radians.

a. [2 points] Find the coordinates of the point Q .

For full credit, each coordinate should be exact and simplified as much as possible.

Solution: Because the radius is 5 and the angle counterclockwise from the positive x -axis to Q is $\frac{\pi}{6}$, the coordinates of point Q are $(5 \cos(\frac{\pi}{6}), 5 \sin(\frac{\pi}{6}))$. Recall that $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ and $\sin(\frac{\pi}{6}) = \frac{1}{2}$. Thus the coordinates are $(\frac{5\sqrt{3}}{2}, \frac{5}{2})$.

Answer: The coordinates of Q are $(\underline{\frac{5\sqrt{3}}{2}}, \underline{\frac{5}{2}})$.

b. [2 points] Find the coordinates of the point P in terms of ϕ .

Solution: Because the radius is 5 and the angle counterclockwise from the positive x -axis to P is $\phi + \frac{\pi}{6}$, the coordinates of the point P are $(5 \cos(\phi + \frac{\pi}{6}), 5 \sin(\phi + \frac{\pi}{6}))$.

Answer: The coordinates of P are $(\underline{5 \cos(\phi + \frac{\pi}{6})}, \underline{5 \sin(\phi + \frac{\pi}{6})})$.

c. [2 points] Find the length of the path from Q to P counterclockwise along the circle (the upper path shown in **bold** in the diagram above). Give your answer in terms of ϕ .

Solution: Recall that arc length is equal to $r\theta$, where r is the radius of the circle and θ is the angle (measured in radians). In this case, the radius is 5 and the angle is ϕ , so the arc length from Q to P is 5ϕ .

Answer: $\underline{5\phi}$

d. [5 points] The length of the counterclockwise path along the circle from the point R to the point $(5, 0)$ (the lower path shown in **bold** in the diagram above) is 11 units. Find the coordinates of the point R . For full credit, show your work and give decimal approximations rounded to the nearest 0.01 unit rather than exact answers.

Solution: First, we find the angle spanned by this arc. The angle satisfies $11 = 5\theta$, so $\theta = 11/5 = 2.2$. Because R is at an angle of θ measured clockwise from the positive x -axis, the coordinates of R are $(5 \cos(-2.2), 5 \sin(-2.2)) \approx (-2.94, -4.04)$.

Answer: The coordinates of R are $(\underline{-2.94}, \underline{-4.04})$.

e. [2 points] Based on the diagram above, which of the following statements are true?

Circle ALL of the statements that are true.

Circle NONE OF THESE if none of the statements are true.

Solution: We use the definitions of sine and cosine to compare the values.

- $\cos(\frac{\pi}{6}) > \cos(\phi + \frac{\pi}{6})$
 $0 > \cos(\phi + \frac{\pi}{2})$
 NONE OF THESE
 $\cos(\phi) > \cos(\frac{\pi}{6})$
 $\sin(\phi + \frac{\pi}{6}) > 0$