6. [11 points] For each equation below, solve EXACTLY for the specified variable. *Show your work step-by-step and write your answers in* **exact form** *in the answer blanks provided.*

a. [4 points] $12.1e^{0.15p} = 0.78(0.9)^p$

Solution: We take the natural logarithm of both sides of the equation and then apply basic logarithm properties to solve for p.

(Take the natural logarithm of both sides.)	$\ln(12.1e^{0.15p}) = \ln(0.78(0.9)^p)$
(Apply basic properties of logarithms.)	$\ln(12.1) + \ln(e^{0.15p}) = \ln(0.78) + \ln(0.9^p)$
(Use additional log properties.)	$\ln(12.1) + 0.15p = \ln(0.78) + p\ln(0.9)$
(Isolate p on one side of the equation.)	$0.15p - p\ln(0.9) = \ln(0.78) - \ln(12.1)$
(Factor out p.)	$p(0.15 - \ln(0.9)) = \ln(0.78) - \ln(12.1)$
(Divide to solve for p.)	$p = \dfrac{\ln(0.78) - \ln(12.1)}{0.15 - \ln(0.9)}$

Answer: $p = \dfrac{\ln(0.78) - \ln(12.1)}{0.15 - \ln(0.9)} = \dfrac{\ln(0.78/12.1)}{0.15 - \ln(0.9)}$

b. [4 points] $\dfrac{\ln(z^7) - \ln(z^4)}{\ln(50)} = 5$

Solution:

(Multiply both sides of the equation by $\ln(50)$.)	$\ln(z^7) - \ln(z^4) = 5\ln(50)$
(Use a basic property of logarithms.)	$\ln\left(\dfrac{z^7}{z^4}\right) = 5\ln(50)$
(Simplify.)	$\ln(z^3) = 5\ln(50)$
(Use the definition of ln (or exponentiate).)	$z^3 = e^{5\ln(50)} = \left(e^{\ln(50}\right)^5 = 50^5$
(Solve for z.)	$z = 50^{5/3}$

Note: One alternate approach is to simplify the left side of the equation as follows: $\ln(z^7) - \ln(z^3) = 7\ln(z) - 4\ln(z) = 3\ln(z)$. Then $\ln(z) = \frac{5\ln(50)}{3}$ and $z = e^{\frac{5\ln(50)}{3}} = 50^{5/3}$.

Answer: $z =$ $50^{5/3}$ **or** $e^{5\ln(50)/3}$

c. [3 points] $\ln(10e^{-5n}) = 3n + 2$

Solution: Using basic properties of logarithms, we can first simplify the left side of the equation as follows:

$$\ln(10e^{-5n}) = \ln(10) + \ln(e^{-5n}) = \ln(10) - 5n.$$

Then we can solve for n.

$$\ln(10) - 5n = 3n + 2$$
$$\ln(10) - 2 = 3n + 5n = 8n$$
$$\frac{\ln(10) - 2}{8} = n$$

Answer: $n = \dfrac{\ln(10) - 2}{8}$