

7. [12 points] Last winter, Mollie Mole kept very careful records of her dwindling supply of earthworms. She had 450 grams of earthworms at the beginning of the winter, and 23.5% of her earthworm supply was eaten during the first 10 days of winter.

For this problem, you must find your answers algebraically and show each step carefully.

- a. [2 points] Do not round your answers.

How many grams of earthworms did Mollie eat during the first 10 days of last winter?

Solution: If she ate 23.5% of her earthworm supply of 450 grams during the first 10 days of last winter, she ate $0.235(450) = 105.75$ grams.

Answer: 105.75 grams

How many grams of earthworms were left in Mollie's supply after the first 10 days of last winter?

Solution: She had $450 - 105.75 = 344.25$ grams of earthworms left after the first 10 days of last winter.

Answer: 344.25 grams

Let $W(d)$ be the number of grams of earthworms in Mollie's supply d days after the start of last winter.

- b. [4 points] Assuming that Mollie's supply of earthworms decreased exponentially during the first 10 days of last winter, find a formula (in *exact form*) for $W(d)$ for $0 \leq d \leq 10$.

Solution: Since $W(d)$ is exponential and $W(0) = 450$, there is a constant b so that $W(d) = 450b^d$. We know that $W(10) = (1 - 0.235)450 = 0.765(450)$, so $0.765(450) = 450b^{10}$. Thus $b^{10} = 0.765$ and $b = 0.765^{1/10} = 0.765^{0.1}$.

Answer: $W(d) =$ $450(0.765)^{d/10}$ or $450 \cdot 0.765^{0.1d}$

- c. [1 point] According to your formula above, by what percent did Mollie's supply of earthworms decrease each day during the first 10 days of last winter?

Solution: The daily change of her supply is $b - 1 = 0.765^{0.1} - 1 \approx -0.0264 = -2.64\%$, so her supply decreased each day by $1 - 0.765^{0.1} \approx 0.0264 = 2.64\%$.

Answer: $1 - 0.765^{0.1} \approx 2.64\%$

- d. [5 points] After the first 10 days, for the rest of last winter, Mollie's remaining supply of earthworms decreased by 6.5% each day. How many total days of winter had passed when her supply dropped below 5 grams? Remember to find your answer algebraically, showing each step carefully. Then round to the nearest day.

Solution: At day 10, she has 344.25 grams. If her supply decreases 6.5% each day, then t days after the first 10 days, she has $344.25(0.935)^t$ grams. We solve for t in the equation $344.25(0.935)^t = 5$. We have $0.935^t = \frac{5}{344.25}$, so $\ln(0.935^t) = \ln\left(\frac{5}{344.25}\right)$. Thus

$$t \ln(0.935) = \ln(5) - \ln(344.25) \quad \text{and therefore} \quad t = \frac{\ln(5) - \ln(344.25)}{\ln(0.935)}.$$

So, her supply drops below 5 grams after $10 + \frac{\ln(5) - \ln(344.25)}{\ln(0.935)}$ or about 73 days of winter.

Note: Alternatively, we note that for $d > 10$, a formula for $W(d)$ is given by $W(d) = 344.25(0.935)^{d-10}$. We can then solve for d in $344.25(0.935)^{d-10} = 5$.

Answer: 73 days