8. [12 points] *Note that you do not have to show work on this problem. However, any work or reasoning you do show may be considered for partial credit.*

a. [4 points] Suppose h is an odd function and that $(12, -8)$ is a point on the graph of $y = h(t)$. Find the coordinates of two points that must be on the graph of $y = -3h(t+7)$.

Solution: Since $h(t)$ is odd and $(12, -8)$ is a point on the graph of $h(t)$, the point $(-12, 8)$ is also on the graph of $h(t)$. We obtain the graph of $-3h(t + 7)$ from the graph of $h(t)$ by first stretching vertically by a factor of 3, then reflecting across the horizontal axis, and finally shifting left 7 units. This takes the points $(12, -8)$ and $(-12, 8)$ to the points $(5, 24)$ and $(-19, -24)$, respectively.

Answers: $(5, 24)$ and $(-19, -24)$

b. [4 points] Suppose the graph of $y = k(x)$ has $y = 4$ as its only horizontal asymptote and $x = -2$ as its only vertical asymptote. If $g(x) = k(-3x) + 11$, what are the *equations* of the horizontal and vertical asymptotes of the graph of $y = g(x)$?

Solution: The graph of $k(-3x) + 11$ is obtained from the graph of $k(x)$ by compressing horizontally by a factor of $1/3$, then reflecting across the vertical axis, and finally shifting up 11 units. These transformations take the horizontal asymptote $y = 4$ to $y = 15$ and the vertical asymptote $x = -2$ to $x = 2/3$.

horizontal asymptote: $y = 15$ **vertical asymptote:** $x = \frac{2}{3}$

c. [4 points] Suppose the domain of $f(x)$ is the interval $[-4, \infty)$. Find the domain of the function p defined by $p(x) = 5 - f(-2x + 1)$.

Solution: The only transformations involved in obtaining the graph of $y = p(x)$ from the graph of $y-f(x)$ that affect the domain of the function are the horizontal transformations. Because $p(x) = 5 - f(-2x + 1) = 5 - f(-2(x - \frac{1}{2}))$, the horizontal transformations involved are, in order, a horizontal compression by a factor of $1/2$ towards the vertical axis (changing the domain from $[-4, \infty)$ to $[-2, \infty)$), a reflection across the vertical axis (changing the domain from $[-2, \infty)$ to $(-\infty, 2]$), and, finally, a shift to the right by $1/2$ unit (taking the domain to $(-\infty, \frac{5}{2}]$).

Answer: $(-\infty, \frac{5}{2}]$

9. [5 points] An exponentially growing population of mice triples in size every 120 days. How long does it take this population to increase by 400%?
(*Show your work step-by-step, and give your answer in exact form.*)

Solution: Let P_0 be the initial population of mice, and let $P(t)$ be the population of the mice after t days. Then, since the population is growing exponentially there is a growth factor b so that $P(t) = P_0 b^t$. Since the population triples every 120 days, we know that $P(120) = 3P_0$ so $3P_0 = P_0 b^{120}$. Hence $3 = b^{120}$ and $b = 3^{1/120}$. Because we want to find how long it takes for the population to increase by 400%, we want to find the value of t when $P(t) = 5P_0$, i.e. when $5P_0 = P_0(3^{1/120})^t$. We solve for t.

$$5P_0 = P_0 3^{t/120} \quad \text{so} \quad 5 = 3^{t/120} \quad \text{and} \quad \ln(5) = \ln(3^{t/120}) = \frac{t}{120}\ln(3).$$

Thus it takes $t = \dfrac{120\ln(5)}{\ln(3)}$ days for the population to increase by 400%.

Answer: $\dfrac{120\ln(5)}{\ln(3)}$ **days**