3. [15 points] At a restaurant, the temperature of a bowl of soup S (in °F) t minutes after it is served is given by the function $S = f(t) = 50 + 130e^{-0.07t}$.

a. [2 points] At what temperature was the soup served? Include units.

Solution: $f(0) = 180$°F.

b. [4 points] How long does it take for the temperature of soup to reach 90°F? Find your answer algebraically. Show all your work. Your answer must be exact or accurate up to two decimal places.

Solution: When the temperature of the soup is 90°F, t satisfies the equation

$$90 = 50 + 130e^{-0.07t}$$
$$\frac{4}{13} = e^{-0.07t}$$
$$\ln\left(\frac{4}{13}\right) = \ln(e^{-0.07t})$$
$$\ln\left(\frac{4}{13}\right) = -0.07t.$$
$$t = -\frac{\ln(\frac{4}{13})}{0.07} \approx 16.84 \text{ minutes.}$$

c. [4 points] Does the function f have vertical or horizontal asymptotes? If it does, write down their equations, otherwise write **None**.

Solution:
Equation of the horizontal asymptote: $S = 50$.

Equation of the vertical asymptote: NONE

Problem continues on the next page

The statement of the problem is included here for your convenience.

At a restaurant, the temperature of a bowl of soup S (in °F) t minutes after it is served is given by the function $S = f(t) = 50 + 130e^{-0.07t}$.

d. [5 points] At the same restaurant, the temperature of a cup of coffee C (in °F) t minutes after it is served is given by the function $C = g(t) = 50 + 150\,(0.85)^t$. A bowl of soup and a cup of coffee are served at the same time. How long does it take for the temperature of the soup and the coffee to be equal after they are served? Show all your work. Your answer must be exact or accurate up to two decimal places.

Solution: When the temperature of the coffee and the soup are equal, we have

$$\begin{aligned}
50 + 130e^{-0.07t} &= 50 + 150(0.85)^t \\
130e^{-0.07t} &= 150(0.85)^t \\
\frac{e^{-0.07t}}{(0.85)^t} &= \frac{15}{13}. \\
\left(\frac{e^{-0.07}}{0.85}\right)^t &= \frac{15}{13}. \\
t\ln\left(\frac{e^{-0.07}}{0.85}\right) &= \ln\left(\frac{15}{13}\right) \\
t = \frac{\ln\left(\frac{15}{13}\right)}{\ln\left(\frac{e^{-0.07}}{0.85}\right)} &\approx 1.55 \text{ minutes .}
\end{aligned}$$

Or

$$\begin{aligned}
50 + 130e^{-0.07t} &= 50 + 150(0.85)^t \\
130e^{-0.07t} &= 150(0.85)^t \\
\ln(130e^{-0.07t}) &= \ln(150(0,85)^t. \\
\ln(130) - 0.07t &= \ln(150) + t\ln(0.85). \\
t(-\ln(0.85) - 0.07) &= \ln(150) - \ln(130). \\
t = \frac{\ln(150) - \ln(130)}{-\ln(0.85) - 0.07} &\approx 1.55 \text{ minutes .}
\end{aligned}$$