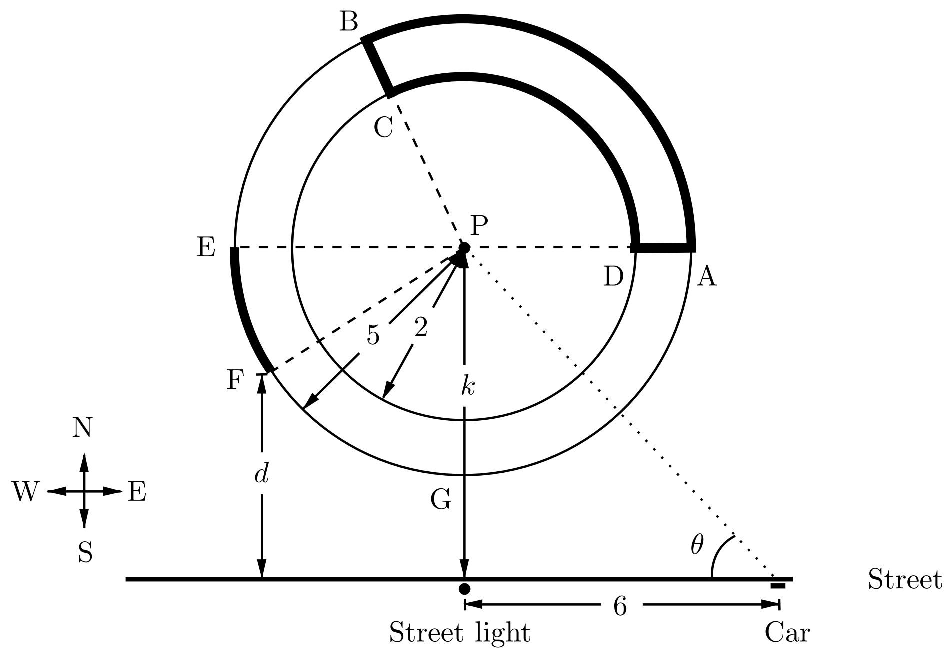
6. [9 points] At a park, there are two circular tracks that are centered at a common flagpole (at point P). The two tracks have radii 2 and 5 km respectively (see the figure below). A street that runs in the east-west direction is located k kilometers south of the flagpole.



a. [4 points] Albert decides to run on the tracks starting at the point A on the east end of the longer track . He runs along the longer track counterclockwise until he reaches point B. Then he runs from point B towards the flagpole until point C on the shorter track. He continues clockwise along the shorter track until point D. From there, he runs east to point A (see the bolded path in the figure). If the distance Albert ran along the longer track between the points A and B is 7 km, what is the total distance he ran?

Solution: Since Albert ran 7 km along the longer track between A and B with radius 5 km, then the arc length formula $s = r\theta$ yields angle $BPA = \frac{s}{r} = \frac{7}{5}$ radians. This angle can be used to find the length of the arc CD in the shorter track with radius 2

$$\text{length of arc } CD = r(\text{angle } BPA) = 2\left(\frac{7}{5}\right) = \frac{14}{5} \text{ kilometers.}$$

Since the distance between B and C and the distance between D and A are both $5-2=3$ km, the total distance ran by Albert is $3+3+7+\frac{14}{5} = 15.8$ km.

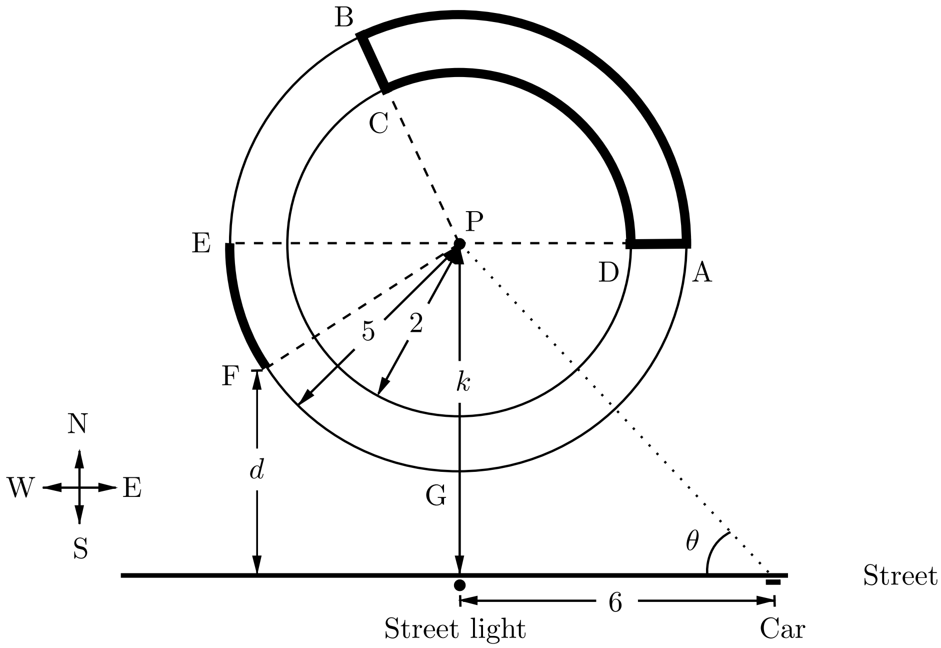
b. [3 points] John starts running at point E, which is the furthest point directly west of the flagpole on the longer track. He plans to run on the track in the counterclockwise direction to the point G, which is directly south of the flagpole. He stops at point F which is a third of the way between point E and G on the track. What is John's distance d (in kilometers) to the street at this point? Your answer may depend on k.

Solution: Since F is a third of the way from E to G, the angle EPF is $\frac{\pi}{6}$. This implies that the distance from F to the line EP is $5\sin(\frac{\pi}{6}) = \frac{5}{2}$. Thus, $d = k - \frac{5}{2}$ km.

Problem continues on the next page

The statement of the problem is included here for your convenience.

At a park, there are two circular tracks that are centered at a common flagpole (at point P). The two tracks have radii 2 and 5 km respectively (see the figure below). A street that runs in the east-west direction is located k kilometers south of the flagpole.



c. [2 points] Directly south of the flagpole, there is a street light on the street. A car is parked 6 km from the streetlight along the street, and the line connecting the car with the flagpole makes an angle θ with the street (see the figure). Find a formula for the distance k (in kilometers) between the flagpole and the street light in terms of θ.

Solution: Consider the right-angled triangle whose vertices are the car, the street light and P. In that triangle, $\tan(\theta) = \frac{k}{6}$, so $k = 6\tan(\theta)$ km.