8. [10 points] The Richter scale is a function r that takes as input the amount of energy E (in kJ) released in an earthquake, and outputs a number. The function r can be given by the formula

$$r(E) = \frac{2}{3}\log\left(\frac{E}{E_0}\right).$$

a. [5 points] An earthquake that releases $63,000$ kJ of energy is assigned the number 2 by the Richter scale. What is the value of E_0? Find your answer algebraically. Show all your work.

Solution: Since an earthquake that releases $63,000$ kJ of energy is assigned the number 2, we have

$$\begin{aligned}
2 &= \frac{2}{3}\log\left(\frac{63,000}{E_0}\right)\\
3 &= \log\left(\frac{63,000}{E_0}\right).\\
10^3 &= 10^{\log\left(\frac{63,000}{E_0}\right)}\\
1000 &= \frac{63,000}{E_0}\\
E_0 &= 63.
\end{aligned}$$

b. [5 points] Let E_A and E_B be the energy (in kJ) released during Earthquake A and Earthquake B respectively. Suppose that the amount of energy released during Earthquake A was 1000 times the amount of energy released during Earthquake B. What is $r(E_A) - r(E_B)$? Simplify as much as possible. Your answer should not involve any of the constants E_A or E_B.

Solution:

$$\begin{aligned}
r(E_A) - r(E_B) &= \frac{2}{3}\log\left(\frac{E_A}{E_0}\right) - \frac{2}{3}\log\left(\frac{E_B}{E_0}\right)\\
&= \frac{2}{3}\log\left(\frac{E_A}{E_B}\right)\\
&= \frac{2}{3}\log\left(\frac{1000E_B}{E_B}\right)\\
&= \frac{2}{3}\log(1000) = \frac{2}{3}(3) = 2.
\end{aligned}$$