8. [10 points] The Richter scale is a function $r$ that takes as input the amount of energy $E$ (in kJ) released in an earthquake, and outputs a number. The function $r$ can be given by the formula

$$r(E) = \frac{2}{3} \log \left( \frac{E}{E_0} \right).$$

a. [5 points] An earthquake that releases 63,000 kJ of energy is assigned the number 2 by the Richter scale. What is the value of $E_0$? Find your answer algebraically. Show all your work.

**Solution:** Since an earthquake that releases 63,000 kJ of energy is assigned the number 2, we have

$$2 = \frac{2}{3} \log \left( \frac{63,000}{E_0} \right)$$

$$3 = \log \left( \frac{63,000}{E_0} \right).$$

$$10^3 = 10^{\log \left( \frac{63,000}{E_0} \right)}$$

$$1000 = \frac{63,000}{E_0}$$

$$E_0 = 63.$$  

b. [5 points] Let $E_A$ and $E_B$ be the energy (in kJ) released during Earthquake A and Earthquake B respectively. Suppose that the amount of energy released during Earthquake A was 1000 times the amount of energy released during Earthquake B. What is $r(E_A) - r(E_B)$? Simplify as much as possible. Your answer should not involve any of the constants $E_A$ or $E_B$.

**Solution:**

$$r(E_A) - r(E_B) = \frac{2}{3} \log \left( \frac{E_A}{E_0} \right) - \frac{2}{3} \log \left( \frac{E_B}{E_0} \right)$$

$$= \frac{2}{3} \log \left( \frac{E_A}{E_B} \right)$$

$$= \frac{2}{3} \log \left( \frac{1000E_B}{E_B} \right)$$

$$= \frac{2}{3} \log(1000) = \frac{2}{3}(3) = 2.$$