4. [9 points] The $\mathcal{M}$-scale M of an object in outer space with diameter D, in thousands of miles, is given by

$$M = f(D) = 2 + 11.5 \log\left(\frac{D}{d_0}\right)$$

where d_0 is a positive constant.

a. [4 points] If the $\mathcal{M}$-scale of a planet is 10, what is its diameter? Solve algebraically showing all your steps. Your answer may depend on the constant d_0.

Solution:

$$\begin{aligned} 2 + 11.5 \log\left(\frac{D}{d_0}\right) &= 10 \\ 11.5 \log\left(\frac{D}{d_0}\right) &= 8 \\ \log\left(\frac{D}{d_0}\right) &= \frac{8}{11.5} \\ \frac{D}{d_0} &= 10^{\frac{8}{11.5}} \\ D &= 10^{\frac{8}{11.5}} d_0 \end{aligned}$$

b. [5 points] Let D_B and D_M be the diameters of two planets, planet Blue and planet Maize, respectively. If the diameter of planet Blue is double the diameter of planet Maize, then what is the difference between $\mathcal{M}$-scale values of planet Blue and planet Maize? Show all your computations step by step. *Simplify your answer as much as possible.*

Solution:

$$\begin{aligned} M_{Blue} - M_{Maize} &= 2 + 11.5 \log\left(\frac{D_B}{d_0}\right) - \left(2 + 11.5 \log\left(\frac{D_B}{d_0}\right)\right) \\ &= 11.5 \log\left(\frac{D_B}{d_0}\right) - 11.5 \log\left(\frac{D_M}{d_0}\right) \\ &= 11.5 \left(\log\left(\frac{D_B}{d_0}\right) - \log\left(\frac{D_M}{d_0}\right)\right) \\ &= 11.5 \log\left(\left(\frac{D_B}{d_0}\right)\left(\frac{d_0}{D_M}\right)\right) = 11.5 \log\left(\frac{D_B}{D_M}\right) \\ &= 11.5 \log\left(\frac{2D_M}{D_M}\right) = 11.5 \log(2) \end{aligned}$$