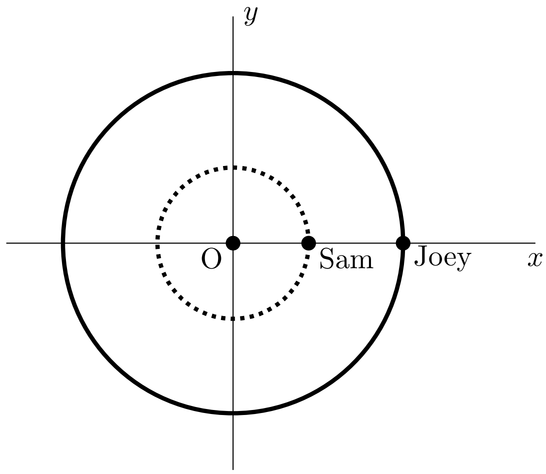
8. [8 points] Three friends decide to go to a circular running track. Joey and Sam will run on the track and Holly will be timing them. Joey runs on the largest circle which has radius 250 meters. Sam runs on the inner most circle which has radius 100 meters.



The run will last 5 minutes, timed by Holly. At Holly's signal, Joey and Sam will both start from the most eastern point of their circle. Assume that both Joey and Sam run at a constant velocity at all times during the 5 minutes and that they both run in the counterclockwise direction.

a. [2 points] Holly notices that it takes Joey 4 minutes to run around the track once. What is the angle θ, in **radians**, that Joey forms with the starting position at the end of the 5 minute run?

Solution: Every minute Joey runs a quarter of the circumference of his track. Hence at the end of 5 minutes, he would have covered an angle of $\frac{5}{2}\pi$ radians.

b. [4 points] On the other hand, Holly records that in one minute Sam covers $\dfrac{7}{8}$ of her track. Imagine that the origin of a coordinate system is placed at the center of the tracks, and that the x-axis is the west-east direction, whilst the y-axis is the south-north direction, as shown in the diagram. Find the coordinates of Sam's final position at the end of the 5 minutes.

Solution: After 5 minutes, Sam covers an angle of $5\left(\frac{7}{8}\right)(2\pi) = \frac{35}{4}\pi$ radians. Hence the coordinates of her final position on the circle of radius 100 are $\left(100\cos\left(\frac{35}{4}\pi\right), 100\sin\left(\frac{35}{4}\pi\right)\right) = (-50\sqrt{2}, 50\sqrt{2})$

c. [2 points] Compute the distance ran by Sam during the 5 minutes of the run.

Solution: At the end of the race, Sam ran a distance of $s = 100\left(\frac{35}{4}\pi\right) = 875\pi$ meters.