

3. [14 points] At a wildlife sanctuary in central Africa, conservationists are carefully monitoring the population of various species of animals. For the following parts, write your answers *in the spaces provided*. Your answers for this problem can either be exact, or accurate to three decimal places.
- a. [3 points] On January 1, 2008, the population of lions in the sanctuary was estimated to be 850, and was decreasing exponentially at a continuous rate of 25% each year. Find a formula for the population  $L(t)$  of lions in the sanctuary  $t$  years after January 1, 2008. **You do not need to show any work for this part.**

$$L(t) = \underline{\hspace{2cm} 850e^{-0.25t} \hspace{2cm}}$$

- b. [5 points] On the other hand, the number of elephants in the sanctuary increased by 60% every 7 years. Let  $E(t)$  be the number of elephants in the sanctuary  $t$  years after January 1, 2008. What is the (annual) continuous growth rate of the function  $E$ ? You should carefully **show your work** for this part.

**Solution:** The function  $E(t)$  is exponential, so we have  $E(t) = ae^{kt}$  for some constants  $a$  and  $k$ . We know that  $E(7) = 1.6a$ , so:

$$\begin{aligned} 1.6a &= ae^{7k} \\ 1.6 &= e^{7k} \\ 7k &= \ln 1.6 \\ k &= \frac{1}{7} \ln 1.6 \end{aligned}$$

The continuous growth rate of  $E$  is  $\underline{\hspace{2cm} \frac{\ln(1.6)}{7} \hspace{2cm}}$  per year.

For the following parts, **you do not need to show any work**, but **you can receive partial credit for work shown if your final answer is incorrect**.

- c. [3 points] Let  $B(m) = 60(3)^{0.5m-1}$  be the number of buffalo in the sanctuary  $m$  months after July 15, 2016. What is the (monthly) continuous growth rate of the function  $B$ ?

The continuous growth rate of  $B$  is  $\underline{\hspace{2cm} \ln(3^{0.5}) \hspace{2cm}}$  per month.

- d. [3 points] Let  $H(y)$  be the total value of donations received by the sanctuary's governing organization (in thousands of dollars)  $y$  years after July 15, 2016. The function  $H$  is exponential, with continuous growth rate  $e^{0.77}$ . What is the annual percentage growth rate of the function  $H$ ?

The annual percentage growth rate of  $H$  is  $\underline{\hspace{2cm} e^{(e^{0.77})} - 1 \hspace{2cm}}$