5. [9 points] At low temperatures, pure tin can deteriorate and become brittle. Martín was recently awarded a tin medal, which he received in the mail; unfortunately, it had already begun to degrade by the time he had received it.

Let $P(t)$ be the fraction of the medal that has degraded t days after Martín first opened the package containing the medal, which is given by the formula:

$$P(t) = \frac{1}{1 + 4(2^{-kt})}$$

where $k > 0$ is a constant.

a. [2 points] What fraction of the medal had already degraded at the time Martín first opened the package? **Circle** your answer from the options below. If none of the options are correct, circle NONE OF THESE.

1 $\boxed{\frac{1}{5}}$ $\frac{1}{9}$ NONE OF THESE

b. [2 points] Let $U(t)$ be the fraction of the medal that has not yet degraded t days after Martín first opened the package. Find a formula that expresses $U(t)$ as a (combination of) transformation(s) of $P(t)$. **You do not need to show any work for this part**, but you should write your answer *in the space provided*.

$U(t) =$ $1 - P(t)$

c. [5 points] Exactly 4 days after first opening the package, Martín finds that the fraction of the metal that has degraded is now 0.95. Set up an equation representing this fact, and use this to find k. Your answer must be found *algebraically*, and must be **exact**. You should carefully **show your work** for this part, and write your answer *in the space provided*.

Solution: We have $P(4) = 0.95$, which gives us:

$$\begin{aligned}
0.95 &= \frac{1}{1 + 4(2^{-4k})} \\
1 + 4(2^{-4k}) &= \frac{1}{0.95} \\
4(2^{-4k}) &= \frac{1}{0.95} - 1 \\
2^{-4k} &= \frac{1}{4}\left(\frac{1}{0.95} - 1\right) \\
-4k \ln 2 &= \ln\left(\frac{1}{4}\left(\frac{1}{0.95} - 1\right)\right) \\
k &= -\frac{1}{4 \ln 2} \ln\left(\frac{1}{4}\left(\frac{1}{0.95} - 1\right)\right)
\end{aligned}$$

$k =$ $-\frac{1}{4 \ln 2} \ln\left(\frac{1}{4}\left(\frac{1}{0.95} - 1\right)\right)$