

7. [11 points] In each of the following parts, you are given an equation in which you must solve for x . Your answers must be **exact** and should be obtained *algebraically*. You should **show all your work**, **step-by-step**, and write your final answers *in the spaces provided*.

a. [3 points] $\ln(3x^7 + 5) = -2$

Solution: We exponentiate both sides and solve for x , which gives us:

$$\begin{aligned} 3x^7 + 5 &= e^{-2} \\ 3x^7 &= e^{-2} - 5 \\ x^7 &= \frac{1}{3}(e^{-2} - 5) \\ x &= \sqrt[7]{\frac{1}{3}(e^{-2} - 5)} \end{aligned}$$

$$x = \sqrt[7]{\frac{1}{3}(e^{-2} - 5)}$$

b. [4 points] $e^{7x} = 5e^{10x}$

Solution: We take the natural logarithm of both sides and use properties of the logarithm to simplify, which gives us:

$$\begin{aligned} \ln(e^{7x}) &= \ln(5e^{10x}) \\ 7x &= (\ln 5) + 10x \\ -3x &= \ln 5 \\ x &= -\frac{1}{3} \ln 5 \end{aligned}$$

$$x = -\frac{\ln 5}{3}$$

c. [4 points] $4(\log(ax))^3 + 8 = 0$, where $a > 0$ is a constant. Your answer for this part may involve a .

Solution: We'll first isolate the $(\log(ax))^3$ on one side, then take a cube root and exponentiate to solve for x :

$$\begin{aligned} (\log(ax))^3 &= -2 \\ \log(ax) &= \sqrt[3]{-2} \\ ax &= 10^{\sqrt[3]{-2}} \\ x &= \frac{1}{a} 10^{\sqrt[3]{-2}} \end{aligned}$$

$$x = \frac{1}{a} 10^{\sqrt[3]{-2}}$$