8. [10 points] Twissell is attempting to determine the depth of a lake at various points by lowering a sensor to the bottom of the lake and measuring the intensity of the sun's light at that point. According to his calculations, if the intensity of the light (in lumens) that reaches the sensor is I, then the sensor must be at a depth of $F(I)$ (measured in meters), which is given by the formula:

$$F(I) = -\frac{1}{4}\log\left(\frac{I}{c}\right)$$

where $c > 0$ is a constant. Note that the intensity of light I at all points underwater is positive and smaller than c, so $F(I)$ is positive.

In the following parts, you must **show all your work**, **step-by-step**, and find your answers *algebraically* to receive full credit. Your final answers must be **exact**, and should be written *in the spaces provided*.

a. [4 points] What is the intensity of the light that reaches the sensor when it is 2 meters underwater? Your answer for this part may include the constant c, and should **include units**.

Solution: Two meters below the surface, the intensity I should satisfy:

$$\begin{aligned} 2 &= -\frac{1}{4}\log\left(\frac{I}{c}\right) \\ \log\left(\frac{I}{c}\right) &= -8 \\ \frac{I}{c} &= 10^{-8} \\ I &= c \cdot 10^{-8} \end{aligned}$$

The intensity 2 meters below the surface is $c \cdot 10^{-8}$

b. [6 points] Twissell submerges the sensor at two different points in the lake.

- At the first point, the depth is d meters and the sensor measures the intensity of the sun's light to be $6K$ lumens.
- At the second point, the depth is D meters and the sensor measures the intensity of the sun's light to be K lumens.

How much deeper is the lake at the second point compared to the first point? Your final answer should be simplified so that it does **not** include the constants K *or* c, but should **include units**.

Solution: We know that:

$$\begin{aligned} D - d &= -\frac{1}{4}\log\left(\frac{K}{c}\right) + \frac{1}{4}\log\left(\frac{6K}{c}\right) \\ &= -\frac{1}{4}\left(\log\left(\frac{K}{c}\right) - \log\left(\frac{6K}{c}\right)\right) \end{aligned}$$

which we can simplify using properties of the logarithm to get:

$$\begin{aligned} &= -\frac{1}{4}\left(\log\left(\frac{K}{c} \cdot \frac{c}{6K}\right)\right) \\ &= -\frac{1}{4}\log\left(\frac{1}{6}\right) \end{aligned}$$

$D - d =$ $-\frac{1}{4}\log\left(\frac{1}{6}\right)$