2. [12 points] The giant cockroaches that escaped from Chuck's farm have separated into two groups. One group has settled beside Lake Popeye not far from Chickenville, the other in the caverns at Kentucky Valley. **Please leave your answers in exact form for all parts of this problem**

a. [6 points] Due to the lush vegetation and ample food near Lake Popeye, the population of cockroaches there is growing exponentially, and has increased by 11% in 6 months. How long will it take for the population to triple in size? *To receive full credit, do not assume in your computations that the initial number of cockroaches is a particular value, and don't forget to include units.*

Solution: Let $P(t) = ab^t$ be the population of cockroaches at Lake Popeye. Since the population increased by 11% in 6 months, we plug in $t = 6$ to get

$$ab^6 = 1.11a \implies b = (1.11)^{1/6}.$$

We now want to find the number of months t when the population triples. In equations, we have

$$a(1.11)^{t/6} = 3a \implies (1.11)^{t/6} = 3.$$

We solve this by taking logarithms.

$$\frac{t}{6}\ln 1.11 = \ln 3 \implies t = \frac{6\ln 3}{\ln 1.11}.$$

The population will triple in $\frac{6\ln 3}{\ln 1.11}$ months.

b. [3 points] The cockroaches in Kentucky Valley have not had it so easy. Their population t months after the escape can be modeled by the function

$$K(t) = 72 \cdot (6/7)^{\frac{1}{2}t-1}.$$

Find the *continuous* monthly growth rate of $K(t)$.

The continuous monthly growth rate is $\frac{1}{2}\ln\frac{6}{7}$.

c. [3 points] What is the *annual* (not continuous) growth rate of $K(t)$?

The annual growth rate is $\left(\frac{6}{7}\right)^6 - 1$.