

3. [11 points] Yolko Ono purchases a serving of her favorite TV dinner, *Chuck's Caterpillar Chop and Gravy*, from Crowger's, her local supermarket chain. At home, she heats up the frozen dish in the microwave oven. Right out of the oven, the temperature of the meal is 185 °F. After 5 minutes, the meal cools to 140 °F. If left out on the counter, the meal will eventually cool to room temperature, 68 °F. **Please leave your answers in exact form for all parts of this problem.**

- a. [7 points] Let $M(t) = A + Be^{kt}$ be the temperature of the meal (in degrees Fahrenheit) t minutes after it leaves the oven. Using the information given, find the values of A , B , and k .

Solution: We first solve for A and B using the value of $M(t)$ at $t = 0$ and the limiting value as t tends to infinity.

$$68 = \lim_{t \rightarrow \infty} M(t) = A$$

$$185 = M(0) = A + B = 68 + B$$

$$B = 185 - 68 = 117$$

Now we solve for k using the value of $M(t)$ at $t = 5$.

$$140 = M(5) = 68 + 117e^{k5}$$

$$e^{k5} = \frac{140 - 68}{117} = \frac{72}{117}$$

$$5k = \ln \frac{72}{117} \implies k = \frac{1}{5} \ln \frac{72}{117}$$

$$A = \frac{68}{\hspace{10em}}$$

$$B = \frac{117}{\hspace{10em}}$$

$$k = \frac{\frac{1}{5} \ln \frac{72}{117}}{\hspace{10em}}$$

- b. [4 points] Yolko has poured a cup of hot coffee into a thick mug. The temperature of the coffee (in degrees Fahrenheit) t minutes after she pours the coffee is given by the function $C(t) = 68 + 100e^{-0.05t}$. Yolko has a sensitive beak and wants to drink the coffee when it is at 131 °F. How long does she have to wait before she can drink it?

Solution: We want to find the value of t such that $C(t) = 131$. Using the formula for $C(t)$, we get

$$131 = 68 + 100e^{-0.05t}$$

$$e^{-0.05t} = \frac{131 - 68}{100} = \frac{63}{100}$$

$$t = \frac{\ln \frac{63}{100}}{-0.05} = 20 \ln \frac{100}{63}$$

She will have to wait $20 \ln \frac{100}{63}$ minutes.