Chuck had a strange yet familiar dream on the night he got back to the farm. His path was blocked by a fast-flowing river. Spying a small tree beside the river, Chuck thought about cutting down the tree to make a log bridge. He knows the points $A$ and $B$ are three meters apart, and he knows the angle between the ground and the line from the points $A$ and $B$ to the top of the tree. The tree is $h$ meters tall and makes a right angle with the ground. The figure below depicts the situation.

Please leave your answers in exact form.

a. [4 points] Write expressions for $\tan(40^\circ)$ and $\tan(50^\circ)$ in terms of $d$ and $h$.

$$\tan(40^\circ) = \frac{h}{d}$$

$$\tan(50^\circ) = \frac{h}{d - 3}$$

b. [4 points] Solve the system of equations you got in part (a) to find $h$ in terms of $\tan(40^\circ)$ and $\tan(50^\circ)$ (and not in terms of $d$).

Solution: We can write $h = d \tan(40^\circ) = (d - 3) \tan(50^\circ)$. Hence

$$d(\tan(50^\circ) - \tan(40^\circ)) = 3 \tan(50^\circ),$$

$$d = \frac{3 \tan(50^\circ)}{\tan(50^\circ) - \tan(40^\circ)}.$$

Substituting $d$ into the first equation for $h$, get

$$h = \frac{3 \tan(40^\circ) \tan(50^\circ)}{\tan(50^\circ) - \tan(40^\circ)}.$$