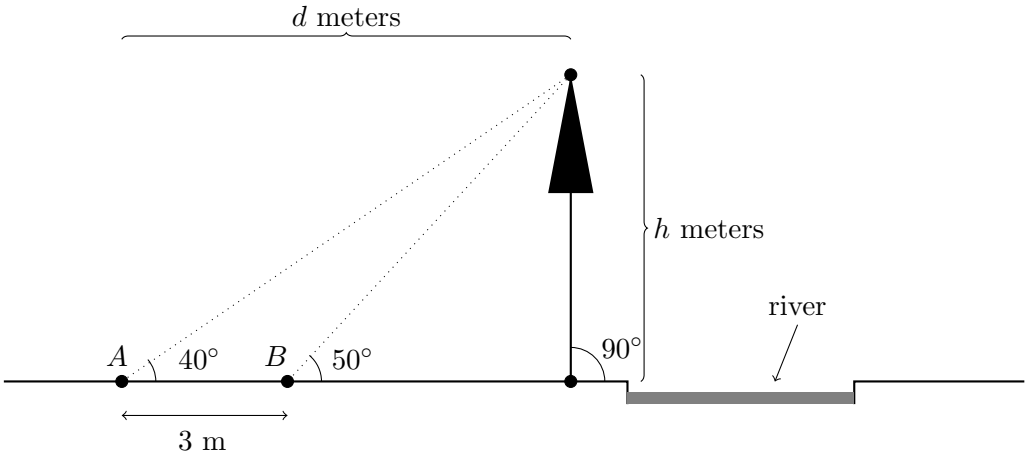
8. [8 points] Chuck had a strange yet familiar dream on the night he got back to the farm. His path was blocked by a fast-flowing river. Spying a small tree beside the river, Chuck thought about cutting down the tree to make a log bridge. He knows the points A and B are three meters apart, and he knows the angle between the ground and the line from the points A and B to the top of the tree. The tree is h meters tall and makes a right angle with the ground. The figure below depicts the situation.



Please leave your answers in exact form.

a. [4 points] Write expressions for $\tan(40°)$ and $\tan(50°)$ in terms of d and h.

$$\tan(40°) = \frac{h}{d}$$

$$\tan(50°) = \frac{h}{d-3}$$

b. [4 points] Solve the system of equations you got in part (a) to find h in terms of $\tan(40°)$ and $\tan(50°)$ (and not in terms of d).

Solution: We can write $h = d\tan(40°) = (d-3)\tan(50°)$. Hence

$$d(\tan(50°) - \tan(40°)) = 3\tan(50°),$$

$$d = \frac{3\tan(50°)}{\tan(50°) - \tan(40°)}.$$

Substituting d into the first equation for h, get

$$h = \frac{3\tan(40°)\tan(50°)}{\tan(50°) - \tan(40°)}.$$

$$h = \frac{3\tan(40°)\tan(50°)}{\tan(50°) - \tan(40°)}$$