4. [9 points] Monica and Rachel recently watched a documentary about earthquakes. They ended up learning a lot about the Richter scale for the magnitude of earthquakes. The Richter rating, R, which measures the magnitude of an earthquake with seismic wave amplitude A is defined as:

$$R = \log\left(\frac{A}{A_0}\right)$$

where A_0 (a positive constant) is the amplitude of the smallest detectable seismic wave. They decide to do some math to get more familiar with the scale. Monica asks Rachel to compute the following:

a. [3 points] An earthquake with a Richter rating of 6 occurred on September 7, 1999 in Athens, Greece. What was the amplitude of its seismic wave? Give your answer in **exact** form. Your answer should be given in terms of A_0.

Solution:

$$\begin{aligned} 6 &= \log\left(\frac{A}{A_0}\right) \\ 10^6 &= \frac{A}{A_0} \\ A &= 10^6 A_0 \end{aligned}$$

Rachel proposes an alternative formula, G, called the "Green rating". The "Green rating" is defined as:

$$G = \frac{1}{2}\left[\log\left(\frac{A}{A_0}\right) + \log\left(\frac{A}{B_0}\right)\right]$$

where B_0 is a constant representing the fixed amplitude of a different seismic wave. Assume that $B_0 > A_0 > 0$.

b. [6 points] Monica notices that the "Green Rating" G is a vertical shift of the original Richter rating R. Determine the shift in terms of A_0 and B_0. Circle UPWARD if it is an upward shift or DOWNWARD if it is a downward shift.

Solution:

$$\begin{aligned} G - R &= \frac{1}{2}\left[\log\left(\frac{A}{A_0}\right) + \log\left(\frac{A}{B_0}\right)\right] - \log\left(\frac{A}{A_0}\right) \\ &= \frac{1}{2}\log\left(\frac{A}{A_0}\right) + \frac{1}{2}\log\left(\frac{A}{B_0}\right) - (\log(A) - \log(A_0)) \\ &= \frac{1}{2}\log(A) - \frac{1}{2}\log(A_0) + \frac{1}{2}\log(A) - \frac{1}{2}\log(B_0) - \log(A) + \log(A_0) \\ &= \frac{1}{2}\log(A_0) - \frac{1}{2}\log(B_0) \\ &= \frac{1}{2}\log\left(\frac{A_0}{B_0}\right) \qquad \text{Since } B_0 > A_0 > 0, \text{ then } \frac{1}{2}\log\left(\frac{A_0}{B_0}\right) < 0 \end{aligned}$$

The vertical shift is UPWARD / $\boxed{\text{DOWNWARD}}$ by $-\frac{1}{2}\log\left(\frac{A_0}{B_0}\right)$ units.