

3. [16 points] For each question below, give your answer(s) in **exact** form where appropriate. The different parts of this problem are not related to each other. **Circle** your final answer for each part.

- a. [4 points] The point $(3, 7)$ is on the graph of $g(x)$. What point must be on the graph of $-3g(2x - 4)$?

Solution: Putting it into the form $-3g(2(x - 2))$, we have that we do a horizontal compression by a factor of $\frac{1}{2}$, followed by a shift right by 2. The vertical transformations are a vertical stretch by a factor of 3, followed by a reflection across the x -axis. Thus, the point we get is

$$\left(\frac{3}{2} + 2, (-3)(7)\right) = \left(\frac{7}{2}, -21\right)$$

- b. [4 points] Find all solutions for x :

$$\ln(x^2 + e^2) = 3$$

Solution: Exponentiating both sides, we have

$$x^2 + e^2 = e^3$$

so

$$x = \pm\sqrt{e^3 - e^2}$$

- c. [4 points] Find the **tripling** time of the exponential function $f(t) = 120e^{0.7t}$, where t is in hours.

Solution: Solving for tripling time is solving

$$3(120) = 120e^{0.7t}$$

From this, we get

$$\ln(3) = 0.7t$$

$$t = \frac{\ln(3)}{0.7} \text{ hours}$$

- d. [4 points] Suppose a farmer can typically grow $B(A)$ bushels of corn on A **acres** of farmland. She starts using a new fertilizer that **doubles** the number of bushels of corn she can grow. Write an expression involving the function B that expresses the number of bushels of corn she can grow on R square meters of farmland if she uses the new fertilizer. (Hint: There are 4046.86 square meters in one acre.)

Solution: Doubling bushel output means we do a vertical stretch by a factor of 2. To get from acres to square meters, we need to horizontally stretch by a factor of 4046.86. Thus, our expression is

$$2B\left(\frac{R}{4046.86}\right)$$