4. [14 points] Roy, the cryptozoologist, has found two more exotic animals on a secret, hidden island! He records the following in his journal:

- **Sorcerer Penguins**: At the time of discovery the population is 3700 penguins, with a continuous yearly decay rate of 4.5%
- **Solar Bears**: $B(t) = 525(0.7)^{2t}$ models the number of solar bears t years after the time of discovery.

Thoughout this problem, give your answers in **exact** form.

a. [3 points] Find a formula for a function $P(t)$ that models the number of Sorcerer Penguins on the island t years after Roy's discovery.

$P(t) =$ $3700e^{-.045t}$

b. [2 points] What is the yearly growth factor of the Sorcerer Penguins?

Yearly growth factor = $e^{-.045}$

c. [3 points] What is the continuous yearly percent decay rate of Solar Bears?

Continuous decay rate = $100(2\ln(0.7))$ %

d. [6 points] Roy also found a third animal on the island, the elusive Iron Lion. The number of Iron Lions t years after Roy's discovery is modeled by the formula

$$I(t) = 50e^{0.5t}$$

Though the initial population of the Iron Lion is much lower, he predicts that they will eventually outnumber the Solar Bears on the island. According to his model, when would that happen? Show all the steps of your calculation, and circle your final answer.

Solution: Setting the two equations equal to each other gives

$$525(0.7)^{2t} = 50e^{0.5t}$$

Taking ln of both sides, we get

$$\ln(525(0.7)^{2t}) = \ln(50e^{0.5t})$$

Applying log rules, we can further reduce to have

$$\ln(525) + 2t\ln((0.7)) = \ln(50) + 0.5t$$

We can then solve directly for t, getting

$$\ln(525) - \ln(50) = (0.5 - 2\ln(0.7))t$$

$$t = \frac{\ln(525) - \ln(50)}{0.5 - 2\ln(0.7)}$$