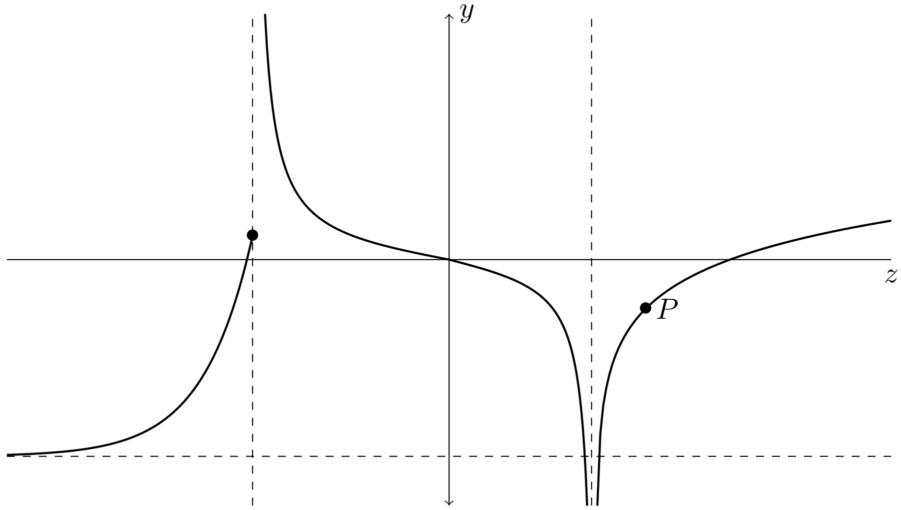
6. [10 points] Below is a graph of the function $S(z)$. The function has a horizontal asymptote at $y = -4$, and vertical asymptotes at $z = -4$ and $z = 3$. The point P is located at the coordinates $(4, -1)$.



a. [4 points]
For $z > 3$, the formula for $S(z)$ is of the form $\log(z-h)+k$. In exact form, find the values of h and k using the fact that $P = (4, -1)$ and the fact that $z = 3$ is a vertical asymptote of $S(z)$.

Solution: $\log(z)$ usually has an asymptote at $z = 0$, so for the asymptote to appear at $z = 3$, we need a horizontal shift to the right by 3 units. This gives us that $h = 3$. With this, we can plug in the point P to get

$$\begin{aligned} -1 &= \log(4-3)+k \\ &= \log(1)+k \\ &= k \end{aligned}$$

$h =$ 3

$k =$ -1

Let $T(z) = 3S(-0.5(z-3)) - 8$.

Note: The next two parts of this problem are about $T(z)$, not about the original function!

b. [4 points] Find the vertical asymptote(s) of $T(z)$. Circle your answer(s).

Solution: The vertical asymptotes of $T(z)$ only depend on the horizontal transformations applied to $S(z)$. We see that the transformations applied are a reflection about the y-axis, followed by a horizontal stretch by a factor of 2, and then a horizontal shift to the right by 3. Applying these to the vertical asymptotes, we get

$$z = 2(-3) + 3 = -3$$

$$z = 2(-(-4)) + 3 = 11$$

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c. [2 points] Find $\lim_{z\to\infty} T(z)$.

Solution: Note that

$$\lim_{z\to\infty} T(z) = \lim_{z\to-\infty} 3S(-0.5(z-3)) - 8 = 3(\lim_{z\to-\infty} S(z)) - 8$$

Since $\lim_{z\to-\infty} S(z) = -4$, we have

$$\lim_{z\to\infty} T(z) = 3(-4) - 8 = -20$$

$\lim_{z\to\infty} T(z) =$ ______ -20 ______

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