8. [8 points] Archaeologists have discovered what seems to be scientific research papers near some dinosaur fossils. The papers talk about the “danger level”, \( L \), of a potential asteroid impact. From what they can read, the formula is given by

\[
L = 3 \log \left( \frac{4M}{k} \right)
\]

where \( M \) is the mass of the asteroid, in kg, and \( k \) is a positive constant. For this problem, leave all your answers in **exact** form.

a. [4 points]

Suppose an asteroid has a danger level of 7.5. What would the mass of the asteroid be? Your answer should include units, and may involve the constant \( k \).

**Solution:**

\[
10^{7.5} = 10^{3 \log \left( \frac{4M}{k} \right)}
\]

\[
= 10^{\log \left( \left( \frac{4M}{k} \right)^3 \right)}
\]

\[
= \left( \frac{4M}{k} \right)^3
\]

Solving then gives \( M = \frac{k}{4} \left(10^{7.5/3} \right) \)

Mass = \( \frac{k}{4} \left(10^{7.5/3} \right) \)

b. [4 points]

Let \( N \) be the danger level of an asteroid of mass \( 12A \) kg, and let \( n \) be the danger level of an asteroid of mass \( 5A \) kg, where \( A \) is a positive constant. Compute \( N − n \). Simplify your answer so that it does **not** include \( k \) or \( A \).

**Solution:** We have \( N = 3 \log \left( \frac{4(12A)}{k} \right) \) and \( n = 3 \log \left( \frac{4(5A)}{k} \right) \). Setting up the difference, we get

\[
N − n = 3 \log \left( \frac{4(12A)}{k} \right) − 3 \log \left( \frac{4(5A)}{k} \right)
\]

\[
= 3 \left( \log \left( \frac{4(12A)}{k} \right) − \log \left( \frac{4(5A)}{k} \right) \right)
\]

\[
= 3 \left( \log \left( \frac{48A}{k} \cdot \frac{k}{20A} \right) \right)
\]

\[
= 3 \log \left( \frac{48}{20} \right)
\]

Where we used a log rule in the third line.

\[
N − n = 3 \log \left( \frac{12}{5} \right)
\]