**8.** [8 points] Archaeologists have discovered what seems to be scientific research papers near some dinosaur fossils. The papers talk about the "danger level", $L$, of a potential asteroid impact. From what they can read, the formula is given by

$$L = 3\log\left(\frac{4M}{k}\right)$$

where $M$ is the mass of the asteroid, in kg, and $k$ is a positive constant. For this problem, leave all your answers in **exact** form.

**a**. [4 points]
Suppose an asteroid has a danger level of 7.5. What would the mass of the asteroid be? Your answer should include units, and may involve the constant $k$.

*Solution:*

$$\begin{aligned} 10^{7.5} &= 10^{3\log\left(\frac{4M}{k}\right)} \\ &= 10^{\log\left(\left(\frac{4M}{k}\right)^3\right)} \\ &= \left(\frac{4M}{k}\right)^3 \end{aligned}$$

Solving then gives $M = \frac{k}{4}(10^{7.5/3})$

Mass = $\frac{k}{4}(10^{7.5/3})$

**b**. [4 points]
Let $N$ be the danger level of an asteroid of mass $12A$ kg, and let $n$ be the danger level of an asteroid of mass $5A$ kg, where $A$ is a positive constant.
Compute $N - n$. Simplify your answer so that it does *not* include $k$ or $A$.

*Solution:* We have $N = 3\log\left(\frac{4(12A)}{k}\right)$ and $n = 3\log\left(\frac{4(5A)}{k}\right)$. Setting up the difference, we get

$$\begin{aligned} N - n &= 3\log\left(\frac{4(12A)}{k}\right) - 3\log\left(\frac{4(5A)}{k}\right) \\ &= 3(\log\left(\frac{4(12A)}{k}\right) - \log\left(\frac{4(5A)}{k}\right)) \\ &= 3(\log\left(\frac{48A)}{k} \cdot \frac{k}{20A}\right)) \\ &= 3\log\left(\frac{48}{20}\right) \end{aligned}$$

Where we used a log rule in the third line.

$N - n =$ $3\log\left(\frac{12}{5}\right)$

 