7. [12 points] A race car is traveling around a circular racetrack at constant speed. It starts at the 3 o’clock position and moves counter-clockwise around a circular track that has radius 600 meters. It takes $2/3$ of a minute for the car to go from the starting point to the point marked $Q$.

(a) [3 points] What is the speed of the car in meters per minute? Note that the given angle is $2\pi/3$ radians.

\begin{solution}
We first need the distance the car has traveled. The car has gone 1/3 of the full circumference of the circle. The circumference is $2\pi R$, where $R = 500, 600, \text{ or } 700$, and so the distance is $2\pi R/3$. Since it took $2/3$ of a minute to do so, the speed is

$$\frac{2\pi R/3}{2/3} = \pi R.$$ 

Depending on your $R$, this will be $500\pi$, $600\pi$, or $700\pi$.
\end{solution}

(b) [4 points] Write a formula for $P(t)$, the $x$-coordinate of the car’s position $t$ minutes after the car leaves the start line, where the center of the track is at the origin. Your answer will be a sinusoidal function, and all constants should be left in exact form.

\begin{solution}
The formula will be

$$P(t) = R\cos(\pi t)$$

(\text{where again } R = 500, 600 \text{ or } 700).
\end{solution}

(c) [5 points] Using your answer to part b, what are the first two positive values of $t$ (in exact form) at which the $x$-coordinate of the car is equal to 100?
Solution: To find the first solution, set $P(t) = 100$ and solve using inverse trig functions:

\[ R \cos(\pi t) = 100 \]

\[ \cos(\pi t) = \frac{100}{R} \quad (100/R \text{ will be } 1/5, 1/6, \text{ or } 1/7) \]

\[ \pi t = \arccos(100/R) \]

\[ t = \frac{\arccos(100/R)}{\pi} \]

To find the second solution, add a period (2 minutes) to the negative of the first solution:

\[ t = -\frac{\arccos(100/R)}{\pi} + 2 \]

(Again, $100/R$ will be $1/5, 1/6,$ or $1/7.$)