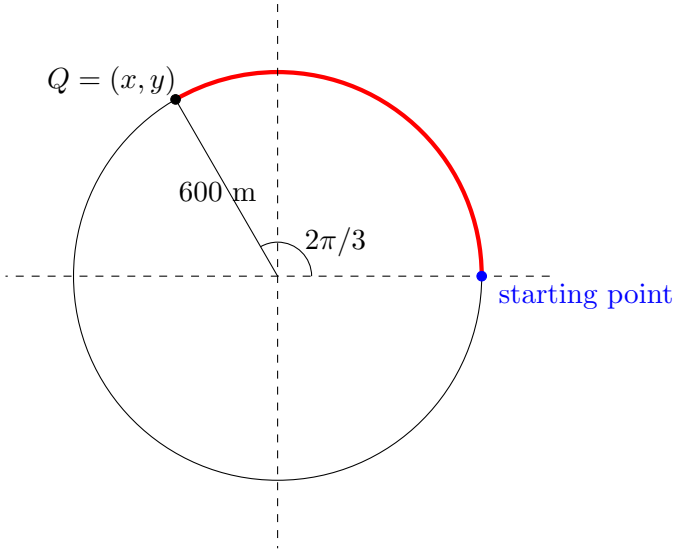
7. [12 points] A race car is traveling around a circular racetrack at **constant** speed. It starts at the 3 o'clock position and moves counter-clockwise around a circular track that has radius 600 meters.

It takes 2/3 of a minute for the car to go from the starting point to the point marked Q.



a. [3 points] What is the speed of the car in meters per minute? Note that the given angle is $2\pi/3$ *radians*.

Solution: We first need the distance the car has traveled. The car has gone 1/3 of the full circumference of the circle. The circumference is $2\pi R$, where $R = 500, 600$, or 700, and so the distance is $2\pi R/3$. Since it took 2/3 of a minute to do so, the speed is

$$\frac{2\pi R/3}{2/3} = \pi R.$$

Depending on your R, this will be 500π, 600π, or 700π.

b. [4 points] Write a formula for $P(t)$, the x-coordinate of the car's position t minutes after the car leaves the start line, where the center of the track is at the origin. Your answer will be a sinusoidal function, and all constants should be left in exact form.

Solution: The formula will be

$$P(t) = R\cos(\pi t)$$

(where again $R = 500, 600$ or 700).

c. [5 points] Using your answer to part **b.**, what are the first two positive values of t (in exact form) at which the x-coordinate of the car is equal to 100?

Solution: To find the first solution, set $P(t) = 100$ and solve using inverse trig functions:

$$\begin{aligned} R\cos(\pi t) &= 100 \\ \cos(\pi t) &= 100/R \qquad (100/R \text{ will be } 1/5,\ 1/6, \text{ or } 1/7) \\ \pi t &= \arccos(100/R) \\ t &= \frac{\arccos(100/R)}{\pi} \end{aligned}$$

To find the second solution, add a period (2 minutes) to the negative of the first solution:

$$t = -\frac{\arccos(100/R)}{\pi} + 2$$

(Again, $100/R$ will be $1/5$, $1/6$, or $1/7$.)