4. [5 points]
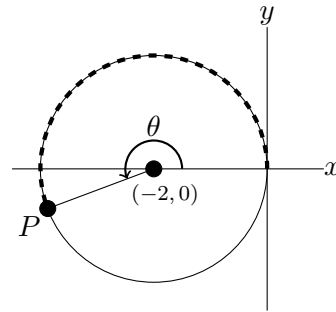
In the diagram at right, a circle of radius 2 is centered at the point $(-2, 0)$. The **bold**, dashed arc going from the origin to the point P has length 7.

(i) Find the exact value of the measure of the angle θ, in radians.

Answer: $\theta = $ $\dfrac{7}{2}$

(ii) Find the (x, y)-coordinates of the point P.

Answer: $(x, y) = \left(2\cos\left(\frac{7}{2}\right) - 2, \ 2\sin\left(\frac{7}{2}\right) \right)$

5. [12 points] Billy Corgi (the lead singer of The Squash) left his pool uncovered when he went on tour. Due to the rainy weather while he was on tour, the volume, in m³, of the water in the pool w weeks after he goes on tour is given by $p(w) = 10e^{0.05w}$.

a. [3 points] Find $p^{-1}(50)$ and interpret your answer in the context of this problem. Show your work. Your answer should be in exact form or correct to two decimal places.

$$10e^{0.05w} = 50$$
$$e^{0.05w} = 5$$
$$0.05w = \ln(5)$$

$$w = \frac{\ln(5)}{0.05}$$

Answer: $p^{-1}(50) = $ $\dfrac{\ln(5)}{0.05} \approx 32.19$

Interpretation:

The volume of the pool is 50 m³ when Billy Corgi has been on tour for $\dfrac{\ln(5)}{0.05}$ weeks.

b. [2 points] What kind of function is the composition $h(w) = \log(p(w))$?

$\log\left(10e^{0.05w}\right)$
$= \log(10) + 0.05w \cdot \log(e)$

(i.) $h(w)$ is linear

ii. $h(w)$ is quadratic

iii. $h(w)$ is exponential

iv. NONE OF THESE

5., continued. *Restated from the previous page.* Billy Corgi (the lead singer of The Squash) left his pool uncovered when he went on tour. Due to the rainy weather while he was on tour, the volume, in m^3, of the water in the pool w weeks after he goes on tour is given by $p(w) = 10e^{0.05w}$.

Recall from the first exam that the volume (in m^3) of Alex Artakis' pool w weeks after the Neverclear tour began is given by $v(w) = 120(0.9775)^w$. Assume that the bands started their tours at the same time.

c. [4 points] is there a time at which Billy's pool has the same volume of water as Alex's? Either find this value, in exact form or correct to two decimal places, showing step-by-step work, or explain why no such time exists.

$$10\,e^{0.05w} = 120\,(0.9775)^w$$

$$e^{0.05w} = 12\,(0.9775)^w$$

ln of both sides:

$$0.05w = \ln\left(12\,(0.9775)^w\right)$$

$$0.05w = \ln(12) + \ln\left(0.9775^w\right)$$

$$0.05w = \ln(12) + w \cdot \ln(0.9775)$$

Subtract $w \cdot \ln(0.9775)$ from both sides

$$0.05w - w \cdot \ln(0.9775) = \ln(12)$$

$$w\left(0.05 - \ln(0.9775)\right) = \ln(12)$$

$$w = \frac{\ln(12)}{0.05 - \ln(0.9775)} \text{ weeks}$$

$$\approx 34.15$$

d. [3 points] Give a practical interpretation of the expression $v^{-1}(p(8))$, or explain why the expression doesn't make sense in the context of the problem. You do **not** need to evaluate this expression.

$v^{-1}(p(8))$ weeks into the tours, Alex's pool will contain the same amount of water as Billy's pool did 8 weeks into the tour.