3. [11 points] Three bacteria colonies, called A, B, and C, are established at the same time. The number of bacteria in these colonies are given by $A(t)$, $B(t)$, and $C(t)$, where t is measured in hours since the colonies were established. The formulas for these functions are

$$B(t) = 500 \cdot 3^{t+1}$$
$$C(t) = 100 \cdot e^{2t}$$

a. [1 point] How many bacteria did Colony B start with?

Answer: $500 \cdot 3 = 1500$

b. [1 point] Which, if any, colonies have a percent growth rate of 200% per hour?
Circle **all** that are correct.

A $\boxed{\text{B}}$ C None

c. [6 points]

(i) Starting from the time the colonies were established, will colonies A and B ever have the same number of bacteria? If so, find the time when this happens, in exact form or rounded to at least two decimal places. If not, briefly explain why not.

Answer *(circle one)*: Yes: $t =$ ______ $\boxed{\text{No}}$ (explain below)

Solution: B starts out with a larger population and grows at a faster rate, so they will never be equal.

(ii) Starting from the time the colonies were established, will colonies A and C ever have the same number of bacteria? If so, find the time when this happens, in exact form or rounded to at least two decimal places. If not, briefly explain why not.

Answer *(circle one)*: $\boxed{\text{Yes:}}$ $t = \dfrac{\ln(2)}{2-\ln(2)}$ No (explain below)

Solution: These will be the same when $A(t) = C(t)$, or $200 \cdot 2^t = 100 \cdot e^{2t}$. Solving for t, we find:

$$\begin{aligned}
200 \cdot 2^t &= 100 \cdot e^{2t} \\
2 \cdot 2^t &= e^{2t} \\
\ln(2 \cdot 2^t) &= \ln(e^{2t}) \\
\ln(2) + t \cdot \ln(2) &= 2t \\
2t - t\ln(2) &= \ln(2) \\
t(2 - \ln(2)) &= \ln(2) \\
t &= \frac{\ln(2)}{2-\ln(2)} \approx 1.885
\end{aligned}$$

Recall: A bacteria colony C has population $C(t)$, where t is measured in hours since the colony was established. The formulas for this function is

$$C(t) = 100 \cdot e^{2t}$$

d. [3 points] Find a formula for $g(P)$, a function that gives the amount of time (in hours) it takes for colony C to reach P bacteria.

Solution: We are being asked to find $C^{-1}(P)$. We do this by solving $P = C(t)$ for t:

$$\begin{aligned} P &= 100e^{2t} \\ P/100 &= e^{2t} \\ \ln(P/100) &= 2t \\ t &= \frac{1}{2}\ln(P/100) \end{aligned}$$

Answer: $g(P) =$ $\frac{1}{2}\ln(P/100)$

4. [7 points] Let $g(x) = 2 \cdot (0.5)^{-3x} - 6$.

a. [5 points] List the transformations you need to apply to the graph of $y = 0.5^x$ to transform it to that of $y = g(x)$. Fill each space with either a number or one of the phrases below, as appropriate. (Leave the second blank empty for reflections.)

Solution: One note: the only order here that matters is that the vertical stretch happens before the shift down.

First, **reflect it across the y-axis** by ______

then, **Compress it horizontally** by 1/3

then, **stretch it vertically** by **2**

then, **shift it down** by **6**

b. [2 points] Give equations for all vertical and horizontal asymptotes of $g(x)$. If there are none, write None.

Answer: Vertical Asymptotes: **None**

Answer: Horizontal Asymptotes: $y = -6$