

6. [11 points] A video is posted online and later goes viral after it is shared by a certain celebrity on a social media platform. 2 hours after it is shared, it has 5 thousand views, and 6 hours after it is shared, it has 10 thousand views.
- a. [2 points] Suppose that the number of views increases at a constant rate of views per hour. Find a formula for $f(t)$, the number of views, in thousands, that the video has t hours after it is shared.

Solution: If the number of views per hour is increasing at a constant rate, then this must be a linear function. We find the slope: $\frac{10 - 5}{6 - 2} = \frac{5}{4}$ and then can either put it in point-slope form or find the vertical intercept of $5/2$.

$$\text{Answer: } f(t) = \frac{5}{4}t + \frac{5}{2} = 1.25t + 2.5$$

- b. [4 points] Suppose instead that the number of views increases at a constant **percent** growth rate, find a formula for $g(t)$, the number of views, in thousands, that the video has t hours after it is shared.

Solution: A constant percent growth rate means that the function is exponential, so it must be of the form $g(t) = ab^t$. We plug in the two points we know: $ab^2 = 5$ and $ab^6 = 10$, and then solve for b and then a .

$$\text{Answer: } g(t) = \left(\frac{5}{2^{1/2}}\right)(2^{1/4})^t$$

- c. [2 points] Suppose that the number of views increases at a constant percent growth rate and M is a number greater than 4. Which of the following numbers is **greater**?
- Let A be the time, in hours, it takes for the number of views to increase from 4 thousand to 12 thousand.
 - Let B be the time, in hours, it takes for the number of views to increase from M thousand to $3M$ thousand.

Answer (Circle one):

A is greater B is greater They are equal Cannot be determined

Solution: We are being asked here about the *tripling time* of an exponential function. By its nature, growing by a constant percent growth rate, it will always take the same amount of time to grow by the same percentage, regardless of what the starting amount is. So the amount of time it takes for an exponential function to triple from 4 to 12 is the same as the amount of time it takes for the same function to triple from M to $3M$.

- d. [3 points] Another video has gone viral, and the number of views for that video increases by 40% in 2 hours. Find the **continuous** hourly percent growth rate of the number of views of this video. Give your answer in exact form or correct to at least two decimal places.

We know $ae^{2k} = 1.4a$, and want to find k as a percent. Dividing both sides by a and then taking \ln of both sides, we get $2k = \ln(1.4)$, so $k = \ln(1.4)/2$. To find this as a percentage, we need to multiply by 100.

Answer: Continuous hourly percent growth rate: $\frac{\ln(1.4)}{2} \cdot 100 \approx 16.82$ %