4. [7 points] On a warm fall day, Schinella decides to walk home from work. Let $d = f(t)$ be the function giving Schinella's distance **from work**, in miles, t minutes after she leaves work.

a. [3 points] Her walk home from work is 3 miles. Schinella wants to write a new function $g(h)$ that gives her distance **from home**, in miles, h **hours** after she leaves work. Write a formula for $g(h)$ in terms of f.

$g(h) =$ $3 - f(60h)$

b. [2 points] Schinella (who is from Canada) wants to write another new function $k(t)$ that gives her distance from work in **kilometers** t minutes after she leaves work. Given that 1 mile is about 1.6 kilometers, circle the correct formula for $k(t)$ below.

$\boxed{1.6f(t)}$ $\quad f(1.6t) \quad \frac{1}{1.6}f(t) \quad f\left(\frac{t}{1.6}\right)$

c. [2 points] Let $c(t)$ be the function that gives the number of episodes of the podcast *Canadaland* that Schinella has listened to in the first t minutes of her walk. Assume that both $c(t)$ and $f(t)$ are invertible. Using those functions or their inverses, write an expression for Schinella's distance from work, in miles, after she's listened to 2.5 episodes of *Canadaland* while walking home.

$f(c^{-1}(2.5))$ miles

5. [13 points]

a. [4 points] A zookeeper has determined that the function $w(t)$ below provides a good model of the weight, in ounces, of a certain kind of snake t years after it hatches.

$$w(t) = -2e^{-(t-16)/5} + 52$$

Find the value of each of the following **as numbers rounded to two decimal places**. Then **briefly interpret** what each quantity means in the context of the problem.

i. $w(0) =$ ≈ 2.93 **Meaning:** When the snake is first born, it weighs approximately 2.93 ounces.

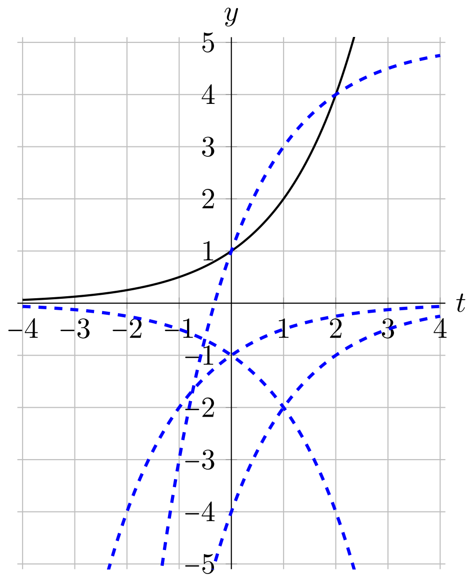
ii. $\lim_{t\to\infty} w(t) =$ 52 **Meaning:** As the snake ages, its weight gets closer and closer to 52 ounces.

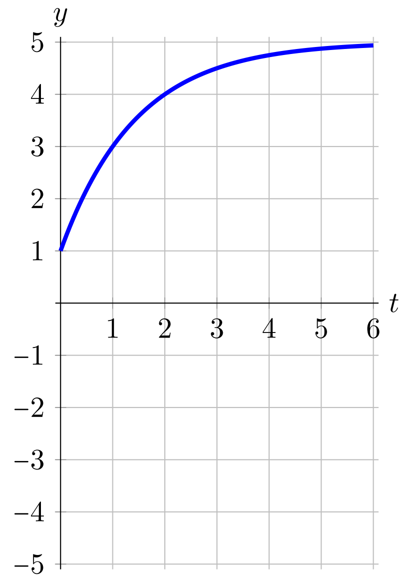
(Problem continues on the next page.)

b. [2 points] The zookeeper also has a model $\ell(t)$ of the length, in feet, of this type of snake t years after it hatches.

$$\ell(t) = -2^{-(t-2)} + 5$$

Using the graph of $y = 2^t$ below as a starting point, sketch a graph of $y = \ell(t)$, for $0 \le t < 6$, on the axes provided to the right.





Solution: The graph of the left shows sketches of the tranformations done, in order, to the original graph as a way of arriving at the final graph. One could also plug in points such as $t = 1$, $t = 2$, etc., to get a sense for the final graph.

c. [5 points] List the transformations you need to apply to the graph of $y = 2^t$ to transform it to that of $y = \ell(t)$. Fill in the first blank with one of the phrases below. Fill in the second blank with a number, "by a factor of" and a number, or N/A for reflections.

SHIFT IT TO THE LEFT	STRETCH IT HORIZONTALLY	REFLECT IT ACROSS THE y-AXIS
SHIFT IT TO THE RIGHT	COMPRESS IT HORIZONTALLY	REFLECT IT ACROSS THE t-AXIS
SHIFT IT UP SHIFT IT DOWN	STRETCH IT VERTICALLY	COMPRESS IT VERTICALLY

First, **Reflect it across the t-axis** by **N/A**

then, **Reflect it across the y-axis** by **N/A**

then, **Shift it to the right** by **2**

then, **Shift it up** by **5**

Note that there are multiple orders this could be listed it. What's important is that the reflection across the t-axis is listed before the shift up; and that the reflection across the y-axis is listed before the shift right. If ether shift is listed *first*, then the direction of the shift would have to be reversed from what is shown here in order to account for the reflection coming second.

Also note that there is a wholly other way to think about this problem. Because $-2^{-(t-2)}+5$ is algebraically equivalent to $-2^{-t}2^4+5$ the horizontal shift is actually equivalent to a vertical stretch by 16. Any students who gave a vertical stretch instead of a horizontal shift would also have received full credit.

d. [2 points] Give equations for all vertical and horizontal asymptotes of $\ell(t)$. If there are none, write NONE.

Vertical Asymptotes: **NONE**

Horizontal Asymptotes: $y = 5$

Solution: Because this is a shifted exponential function, there are no vertical asymptotes, only horizontal ones. We can see that as we plug in larger and larger values of t, that the term $-2^{-(t-2)}$ gets smaller and smaller, approaching 0, because we are raising 2 to higher and higher negative powers. So overall the function will approach 5.

Another way to see this is by looking at how the original asymptote $y = 0$ is transformed under the transformations in the list above. The only one that affects it is the final shift up by 5, make an asymptote of $y = 5$.