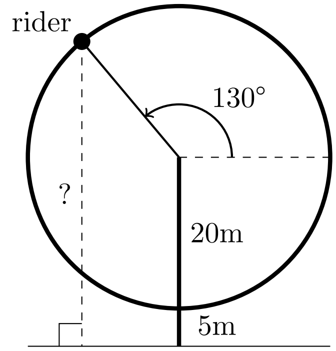
6. [10 points] A diagram showing a Ferris wheel is below. The radius of the Ferris wheel is 20 meters, and the lowest point (where people board) is reached by a small set of stairs and is 5 meters above ground level. We'll consider the following question about this scenario:

When an arm of the Ferris wheel is making an angle of $130°$ *with horizontal, how high is that rider off the ground?*



a. [3 points] Ayisha really likes to do trig problems using a unit circle perspective. She comes up with a correct answer that involves $\sin(130°)$ in her answer. What is Ayisha's answer? *Give your answer in exact form.*

Solution: The vertical distance of the rider above the center of the Ferris wheel would be $\sin(130°)$ *if* we were in the unit circle. But because this circle is scaled up by 20m, their height above the center of the Ferris Wheel level woud be $20 \cdot \sin(130°)$. To get all the way to the ground, we need to add the distance from the Ferris wheel center to the ground, which is 25 meters. This gives us our final answer below.

Another way to think of this is using the formula for a point on a circle of radius r centered at point (x, y). In this case, we are looking for a y-coordinate, so the formula would give us the same thing: $20 \cdot \sin(130°) + 25$.

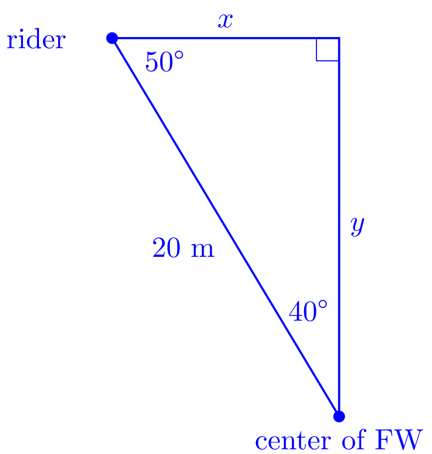
$20\sin(130°) + 25$ meters

b. [2 points] Oh no! Ayisha accidentally had her calculator in radians when she computed her answer. What would she get in that case? How could she recognize right away that her answer was incorrect?

Solution: If you put the expression above into the calculator when your calculator is in radians, then you get approximately 6.398. This is clearly wrong because the rider should be above 25m when they're at $130°$.

c. [4 points] Bruno really likes to do trig problems using right triangles. He comes up with a correct answer that involves $\cos(40°)$. Draw the right triangle Bruno could have been considering and use that triangle to find Bruno's correct expression for the rider's height from the ground. *Give your answer in exact form.*

Solution: Here is a diagram of a triangle extracted from the Ferris wheel picture, with known and unknown lengths and angles shown.



We want to know the value of y in the picture, so we can use either 40° or 50° to find it. Since we're told Bruno used 40°, we'll use that!
Then we know that $\cos(40^\circ) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{y}{20}$. We can solve for y and get $y = 20\cos(40^\circ)$.

(Another way to see is this is to imagine rotating the triangle 90° clockwise. Then we can think of 40° as an angle above the x-axis, and the length labeled y as a horizontal (cosine) measurement in a triangle scaled up to a radius of 20m.)

Adding this value for y to the 25 meter height of the center of the Ferris wheel, we get

$$20\cos(40^\circ) + 25$$

as the riders height off of ground level.

$\underline{\quad 20\cos(40^\circ) + 25 \quad}$ meters.

d. [1 point] Using a calculator, verify that Ayisha's and Bruno's answers agree. That is, find the numerical value of both expressions. *(We're now assuming that Ayisha's calculator is correctly back in degrees!)*

Solution: Both expressions evaluate to approximate 40.32. This also makes sense numerically because it's between 25 and 45; further, visually we can see that it should be closer to 45 than to 25.