3. [6 points] Returning to the scenario in the previous problem: a tank is full of a fixed amount of neon gas and the function $g(t)$ gives the temperature of the gas in the tank T (in °C) as a function of t, where t is measured in minutes since a heating source was turned on. Now we learn further that, for some constant $c < 0$:

$$g(t) = 300 - 250e^{ct}$$

a. [3 points] Assume that the domain for $g(t)$ in this context is $[0, \infty)$. In that case, what is the associated *range* of $g(t)$ and what does this mean in the context of the problem? *Show all relevant work.*

Range: $[50, 300)$

Solution: Note that $g(t)$ is an increasing function since $250e^{ct}$ is a decreasing function if $c < 0$. We have that $g(0) = 300 - 250e^0 = 50$. On the other hand, since $250e^{ct}$ is always positive but approaches 0 as $t \to \infty$, $g(t)$ is always less than 300 but approaches 300 as $t \to \infty$.

Meaning:

Solution: The gas starts at a temperature of 50°C when the heating source is turned on and approaches a temperature of 300°C as time goes on.

b. [3 points] If 30 minutes after the heating source is turned on the temperature of the gas in the tank reaches 200°C, what must be the value of c? *Show all work. Leave your answer in exact form.*

$c = \dfrac{\ln(2/5)}{30}$

Solution: We are given that $g(30) = 200$. That is, $300 - 250e^{30c} = 200$. We need to solve this equation for c:

$$\begin{aligned}
300 - 250e^{30c} &= 200 \\
-250e^{30c} &= -100 \\
e^{30c} &= \frac{-100}{-250} \\
e^{30c} &= \frac{2}{5} \\
30c &= \ln(2/5) \\
c &= \frac{\ln(2/5)}{30}
\end{aligned}$$