

3. [5 points] A mysterious substance decays by 30% every 6 years. Find the half-life of this substance. (Show your work carefully and either give your answer in exact form or round your answer to the nearest 0.01 year.)

Solution: Let $Q(t)$ be the quantity of the mysterious substance in year t and let a be the initial quantity. Then $Q(t) = ae^{kt}$ for some constant k .

Since the substance decays by 30% every 6 years, $Q(6) = 0.7a$ so $0.7a = ae^{6k}$. Then

$$\begin{aligned} 0.7a &= ae^{6k} \\ 0.7 &= e^{6k} \\ \ln(0.7) &= 6k \\ k &= \ln(0.7)/6. \end{aligned}$$

If h is the half-life of the substance, then $Q(h) = 0.5a$, so we have $0.5a = ae^{kh}$. Using the value of k we found above, this gives $0.5a = ae^{h \ln(0.7)/6}$ and we can solve for h .

$$\begin{aligned} 0.5a &= ae^{h \ln(0.7)/6} \\ 0.5 &= e^{h \ln(0.7)/6} \\ \ln(0.5) &= \ln\left(e^{h \ln(0.7)/6}\right) \\ \ln(0.5) &= h \ln(0.7)/6 \\ 6 \ln(0.5)/\ln(0.7) &= h. \end{aligned}$$

So the half-life of this mysterious substance is $\frac{6 \ln(0.5)}{\ln(0.7)}$ (or about 11.66) years.

Answer: $\frac{6 \ln(0.5)}{\ln(0.7)} \approx 11.66$ years

4. [7 points] Consider the function B defined by $B(x) = 15 - e^{-0.001x}$.
- a. [3 points] Let $f(x) = e^x$. Use transformations to find a formula for $B(x)$ in terms of f .

$B(x) = -f(-0.001x) + 15$

- b. [4 points] Find the vertical and horizontal asymptotes of the graph of $y = B(x)$. (If there are no vertical or no horizontal asymptotes, write "NONE" on the appropriate line(s).)

Solution: The function $f(x) = e^x$ from part (a) has no vertical asymptotes and has the horizontal asymptote $y = 0$. The graph of $y = B(x)$ is obtained from that of $y = f(x)$ by first stretching horizontally away from the y -axis by a factor of 1000 then reflecting over both the x - and y - axes and finally shifting up by 15 units. The resulting graph still has no vertical asymptote and has a horizontal asymptote of $y = 15$.

Vertical asymptote(s): None

Horizontal asymptote(s): $y = 15$