3. [5 points] A mysterious substance decays by 30% every 6 years. Find the half-life of this substance. (Show your work carefully and either give your answer in exact form or round your answer to the nearest 0.01 year.)

**Solution:** Let \( Q(t) \) be the quantity of the mysterious substance in year \( t \) and let \( a \) be the initial quantity. Then \( Q(t) = ae^{kt} \) for some constant \( k \).

Since the substance decays by 30% every 6 years, \( Q(6) = 0 \cdot 7a \) so \( 0.7a = ae^{6k} \). Then

\[
\begin{align*}
0.7a &= ae^{6k} \\
0.7 &= e^{6k} \\
\ln(0.7) &= 6k \\
k &= \frac{\ln(0.7)}{6}.
\end{align*}
\]

If \( h \) is the half-life of the substance, then \( Q(h) = 0.5a \), so we have \( 0.5a = ae^{kh} \). Using the value of \( k \) we found above, this gives \( 0.5a = ae^{h \ln(0.7)/6} \) and we can solve for \( h \).

\[
\begin{align*}
0.5a &= ae^{h \ln(0.7)/6} \\
0.5 &= e^{h \ln(0.7)/6} \\
\ln(0.5) &= \ln \left( e^{h \ln(0.7)/6} \right) \\
\ln(0.5) &= h \ln(0.7)/6 \\
6 \ln(0.5)/\ln(0.7) &= h.
\end{align*}
\]

So the half-life of this mysterious substance is \( \frac{6 \ln(0.5)}{\ln(0.7)} \) (or about 11.66 years).

**Answer:** \( \frac{6 \ln(0.5)}{\ln(0.7)} \approx 11.66 \text{ years} \)

4. [7 points] Consider the function \( B \) defined by \( B(x) = 15 - e^{-0.001x} \).

a. [3 points] Let \( f(x) = e^x \). Use transformations to find a formula for \( B(x) \) in terms of \( f \).

\[
B(x) = -f(-0.001x) + 15
\]

b. [4 points] Find the vertical and horizontal asymptotes of the graph of \( y = B(x) \). (If there are no vertical or no horizontal asymptotes, write "NONE" on the appropriate line(s).)

**Solution:** The function \( f(x) = e^x \) from part (a) has no vertical asymptotes and has the horizontal asymptote \( y = 0 \). The graph of \( y = B(x) \) is obtained from that of \( y = f(x) \) by first stretching horizontally away from the \( y \)-axis by a factor of 1000 then reflecting over both the \( x \)- and \( y \)-axes and finally shifting up by 15 units. The resulting graph still has no vertical asymptote and has a horizontal asymptote of \( y = 15 \).

Vertical asymptote(s): **NONE**

Horizontal asymptote(s): **\( y = 15 \)**