

3. [5 points] A mysterious substance decays by 30% every 6 years. Find the half-life of this substance. (Show your work carefully and either give your answer in exact form or round your answer to the nearest 0.01 year.)

*Solution:* Let  $Q(t)$  be the quantity of the mysterious substance in year  $t$  and let  $a$  be the initial quantity. Then  $Q(t) = ae^{kt}$  for some constant  $k$ .

Since the substance decays by 30% every 6 years,  $Q(6) = 0.7a$  so  $0.7a = ae^{6k}$ . Then

$$\begin{aligned} 0.7a &= ae^{6k} \\ 0.7 &= e^{6k} \\ \ln(0.7) &= 6k \\ k &= \ln(0.7)/6. \end{aligned}$$

If  $h$  is the half-life of the substance, then  $Q(h) = 0.5a$ , so we have  $0.5a = ae^{kh}$ . Using the value of  $k$  we found above, this gives  $0.5a = ae^{h \ln(0.7)/6}$  and we can solve for  $h$ .

$$\begin{aligned} 0.5a &= ae^{h \ln(0.7)/6} \\ 0.5 &= e^{h \ln(0.7)/6} \\ \ln(0.5) &= \ln\left(e^{h \ln(0.7)/6}\right) \\ \ln(0.5) &= h \ln(0.7)/6 \\ 6 \ln(0.5)/\ln(0.7) &= h. \end{aligned}$$

So the half-life of this mysterious substance is  $\frac{6 \ln(0.5)}{\ln(0.7)}$  (or about 11.66) years.

**Answer:**  $\frac{6 \ln(0.5)}{\ln(0.7)} \approx 11.66$  years

4. [7 points] Consider the function  $B$  defined by  $B(x) = 15 - e^{-0.001x}$ .
- a. [3 points] Let  $f(x) = e^x$ . Use transformations to find a formula for  $B(x)$  in terms of  $f$ .

$B(x) = -f(-0.001x) + 15$

- b. [4 points] Find the vertical and horizontal asymptotes of the graph of  $y = B(x)$ . (If there are no vertical or no horizontal asymptotes, write "NONE" on the appropriate line(s).)

*Solution:* The function  $f(x) = e^x$  from part (a) has no vertical asymptotes and has the horizontal asymptote  $y = 0$ . The graph of  $y = B(x)$  is obtained from that of  $y = f(x)$  by first stretching horizontally away from the  $y$ -axis by a factor of 1000 then reflecting over both the  $x$ - and  $y$ - axes and finally shifting up by 15 units. The resulting graph still has no vertical asymptote and has a horizontal asymptote of  $y = 15$ .

Vertical asymptote(s): None

Horizontal asymptote(s):  $y = 15$