

5. [12 points] *Note: You do not have to show any work on this page.*

a. [6 points] If  $(2, -6)$  is a point on the graph of  $y = h(x)$ , find a point on the graph of each of the functions below.

(i)  $\left(\underline{\quad 1 \quad}, \underline{\quad -6 \quad}\right)$  is a point on the graph of  $y = h(2x)$ .

*Solution:* To obtain the graph of  $y = h(2x)$  from the graph of  $y = h(x)$  we compress horizontally towards the  $y$ -axis by a factor of  $1/2$ , moving  $(2, -6)$  to the point  $(-1, 6)$ .

(ii)  $\left(\underline{\quad -2 \quad}, \underline{\quad -5 \quad}\right)$  is a point on the graph of  $y = h(-x) + 1$ .

*Solution:* To obtain the graph of  $y = h(-x) + 1$  from the graph of  $y = h(x)$ , we first reflect the graph across the  $y$ -axis (moving the point  $(2, -6)$  to the point  $(-2, -6)$ ) and then shift the resulting graph up by one unit (moving the point  $(-2, -6)$  to the point  $(-2, -5)$ ).

(iii)  $\left(\underline{\quad 3 \quad}, \underline{\quad 18 \quad}\right)$  is a point on the graph of  $y = -3h(x - 1)$ .

*Solution:* To obtain the graph of  $y = -3h(x - 1)$  from the graph of  $y = h(x)$ , we first stretch the graph vertically away from the  $x$ -axis by a factor of 3 (moving the point  $(2, -6)$  to the point  $(2, -18)$ ). Then we reflect this graph across the  $x$ -axis (moving the point  $(2, -18)$  to the point  $(2, 18)$ ). Finally, we shift the resulting graph to the right by one unit (moving the point  $(2, 18)$  to the point  $(3, 18)$ ).

b. [6 points] Some data for functions  $g$  and  $k$  is provided in the table below. Use this data to answer the questions that follow.

$x$	1	2	3
$g(x)$	4	-1	-2
$k(x)$	5	4	1

(i) If  $g(x)$  is an even function, find  $g(-2)$ .

*Solution:* Since  $g$  is even,  $g(-2) = g(2)$  and the table indicates that  $g(2) = -1$ . Hence  $g(-2) = -1$ .

**Answer:**  $g(-2) = \underline{\quad -1 \quad}$

(ii) Let  $m(t) = 2k(-t + 1)$ . Find  $m(-2)$ .

*Solution:* Using the provided formula for  $m(t)$  and the given table, we have  $m(-2) = 2k(-(-2) + 1) = 2k(2 + 1) = 2k(3) = 2(1) = 2$ .

**Answer:**  $m(-2) = \underline{\quad 2 \quad}$

(iii) Let  $n(x) = k(x - 1)$ . If  $n(x)$  is an odd function, find  $k(-3)$ .

*Solution:* Using the given formula (or the implication that the graph of  $y = n(x)$  results from shifting the graph of  $y = k(x)$  to the right one unit), we have  $k(-3) = n(-2)$ . Now  $n$  is odd so  $n(-2) = -n(2)$ . Using the formula provided, we thus find that  $k(-3) = n(-2) = -n(2) = -k(2 - 1) = -k(1) = -5$ . Hence  $k(-3) = -5$ .

**Answer:**  $k(-3) = \underline{\quad -5 \quad}$