6. [13 points] A study of mammals in a particular county in Michigan found that at the time of the study there were N groundhogs and that the population of groundhogs was increasing at a rate of 5% per year. Let $G(t)$ be the number of groundhogs in the county t years after the study.

For full credit on this problem, you must solve for all answers algebraically and show all work step-by-step. Answers should either be in exact form or be given to at least four decimal places.

a. [2 points] Find a formula for $G(t)$.

Answer: $G(t) =$ $N(1.05)^t$.

b. [3 points] Find the *continuous* growth rate of the groundhog population.

Solution: In the two standard forms for exponential functions ($Q = ab^t$ and $Q = ae^{kt}$) we have $b = e^k$ so $\ln b = k$. Here, b is the growth factor and k is the continuous growth rate. In the case of these groundhogs, the annual growth factor is 1.05, so the continuous growth rate is $\ln(1.05)$ (or about 4.8790%) per year.

Answer: $\ln(1.05)$ **(or about** 4.8790%**) per year**

c. [3 points] How long will it take for the number of groundhogs to double?

Solution: If d is the number of years it takes for the number of groundhogs to double, then $G(d) = 2N$. Hence we have $2N = N(1.05)^d$ so $2 = (1.05)^d$. Taking the natural logarithm of both sides of this equation, we find that $\ln(2) = \ln((1.05)^d) = d\ln(1.05)$. Hence $d = \ln(2)/\ln(1.05)$. So, it takes $\ln(2)/\ln(1.05)$ (about 14.2067) years for the number of groundhogs to double.

Answer: $\ln(2)/\ln(1.05)$ **(or about** 14.2067**) years**

d. [5 points] In the same study, it was determined that the number of moles and rabbits in the county t years after the study would be given by the formulas $M(t) = 500(0.99)^t$ and $R(t) = 200e^{0.1t}$, respectively. According to these models, when will the population of rabbits be 50% larger than the population of moles?

Solution: The population of rabbits will be 50% larger than the population of moles when $R(t) = 1.5M(t)$. So we need to find t so that $200e^{0.1t} = 1.5(500(0.99)^t)$. Using the definition and basic properties of the natural logarithm, we find

$$\begin{aligned}
200e^{0.1t} &= 1.5(500(0.99)^t)\\
e^{0.1t} &= 3.75(0.99)^t\\
0.1t &= \ln(3.75(0.99)^t) = \ln(3.75) + \ln((0.99)^t) = \ln(3.75) + t\ln(0.99)\\
0.1t - t\ln(0.99) &= \ln(3.75)\\
t(0.1 - \ln(0.99)) &= \ln(3.75)\\
t &= \ln(3.75)/(0.1 - \ln(0.99))
\end{aligned}$$

Therefore, the population of rabbits will be 50% greater than the population of moles $\ln(3.75)/(0.1 - \ln(0.99))$ (or about 12.0105) years after the study.

Answer: $\ln(3.75)/(0.1 - \ln(0.99))$ **(or about** 12.0105**) years after the study**