**8.** [8 points] In the 1970's, seismologists developed the Moment Magnitude Scale (MMS) to estimate the magnitude of large earthquakes in terms of the energy released. Unlike the Richter scale, which is based on the size of seismic waves, the MMS is based on seismic moments (which represent the energy released in an earthquake). The MMS rating of an earthquake is defined to be

$$S = \frac{2}{3}\log\left(\frac{M}{A}\right)$$

where $M$ is the seismic moment of the quake (in dynes/cm) and $A$ is a positive constant.

**a.** [4 points] Let $S_1$ and $S_2$ represent the MMS ratings of two earthquakes with seismic moments $M_1$ and $M_2$, respectively. Using properties of logarithms, find a formula for $S_2 - S_1$ in terms of $M_1$ and $M_2$. **Simplify your formula as much as possible.**

*Solution:* Using the formula provided and basic properties of logarithms, we have

$$\begin{aligned} S_2 - S_1 &= \frac{2}{3}\log\left(\frac{M_2}{A}\right) - \frac{2}{3}\log\left(\frac{M_1}{A}\right) \\ &= \frac{2}{3}\left(\log\left(\frac{M_2}{A}\right) - \log\left(\frac{M_1}{A}\right)\right) \\ &= \frac{2}{3}\log\left(\frac{M_2/A}{M_1/A}\right) \\ &= \frac{2}{3}\log\left(\frac{M_2}{M_1}\right) \end{aligned}$$

**Answer:** $S_2 - S_1 =$ $\frac{2}{3}\log\left(\frac{M_2}{M_1}\right)$.

**b.** [4 points] The San Francisco earthquake of 1989 had an MMS rating of 6.9 and the Northridge, CA earthquake of 1994 had an MMS rating of 6.7. Based on these ratings, how many times greater than the Northridge seismic moment was the San Francisco seismic moment? (Give your answer in exact form or round to the nearest 0.01.)

*Solution:* Let $M_1$ and $M_2$ be the seismic moments of the Northridge and San Francisco earthquakes, respectively. We are asked to find $M_2/M_1$.
Using the formula we found in part (a) (with $S_2 = 6.9$ and $S_1 = 6.7$), we have

$$\begin{aligned} 6.9 - 6.7 &= \frac{2}{3}\log\left(\frac{M_2}{M_1}\right) \\ 0.2 &= \frac{2}{3}\log\left(\frac{M_2}{M_1}\right) \\ 0.3 &= \log\left(\frac{M_2}{M_1}\right) \\ 10^{0.3} &= \frac{M_2}{M_1} \end{aligned}$$

Therefore, $M_2 = 10^{0.3}M_1$, so the seismic moment of the San Francisco earthquake was $10^{0.3}$ (about 2.00) times greater than that of the Northridge earthquake.

**Answer:** The seismic moment from the San Francisco earthquake was $10^{0.3}$ **(or about** 2.00**)** times greater than the seismic moment of the Northridge earthquake.