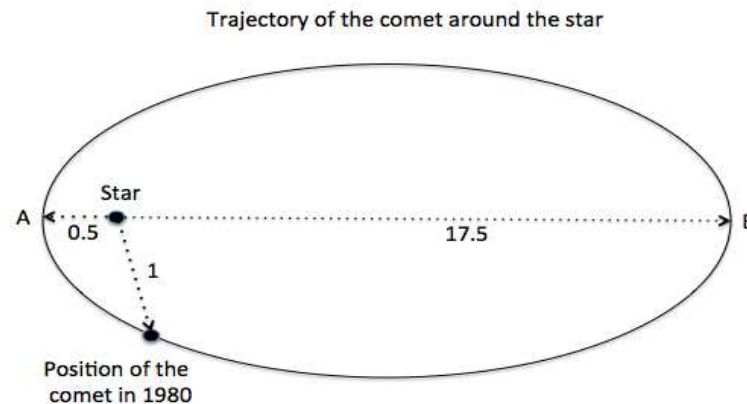


3. [8 points] The trajectory of a comet around a star is shown in the figure below (figure not shown at scale).



The distance between the comet and the star is measured in astronomical units (1 astronomical unit is approximately 150 million kilometers). The following information is known about the trajectory of the comet:

- i) Point A is the closest point, in the comet trajectory, to the star. The distance between the star and point A is 0.5 astronomical units.
- ii) Point B is the farthest point, on the trajectory of the comet, from the star. The distance between the star and point B is 17.5 astronomic units.
- iii) The comet takes 27 years to complete one trip around its trajectory.

Let $C(t)$ be the distance of the comet to the star (in astronomical units) t years after it was discovered in 1980. At the time of its discovery, the comet was 1 astronomical unit away from the star (see the figure).

- a. [6 points] Find the period, amplitude and midline of the periodic function $y = C(t)$.

Solution: Since the comet takes 27 to complete one trip around its trajectory, then the period of $C(t)$ is 27 years.

The maximum distance the comet is ever from the star is 17.5 astronomical units and the minimum distance is 0.5 astronomical units. Hence the amplitude of $C(t)$ is $\frac{17.5 - .5}{2} = 8.5$ astronomical units. The midline is given by the equation $y = \frac{17.5 + 0.5}{2} = 9$.

Period: 27 Amplitude=8.5 and midline: $y=9$.

- b. [2 points] How soon after 2014 should we expect the comet to arrive at the point in the trajectory at which it was discovered in 1980?

Solution: Twenty years after 2014 (in $1980 + 2(27) = 2034$).