

4. [14 points]

a. [4 points]

- i) The point  $P$  lies on a circle of radius three and it corresponds to the angle  $140^\circ$ . Find the coordinates of the point  $P$ . Round your answers to the nearest 0.001.

*Solution:* In this problem, in order to compute the decimal approximation of the coordinates, your calculator must be in degree mode. Hence

$$P = (3 \cos(140^\circ), 3 \sin 140^\circ) = (-2.298, 1.928)$$

- ii) Find angles  $0^\circ < \alpha, \beta < 360^\circ$ , but not equal to  $140^\circ$ , such that

$$\sin \alpha = \sin 140^\circ \quad \alpha = \underline{\hspace{2cm}}.$$

$$\cos \beta = \cos 140^\circ \quad \beta = \underline{\hspace{2cm}}.$$

*Solution:*  $\alpha = 40^\circ$  and  $\beta = 220^\circ$ .

- b. [5 points] Let  $0^\circ < \theta < 45^\circ$ . For each blank space below, determine whether the number on the left is greater than, less than, or equal to the number on the right, and fill in the blank with the symbol  $>$ ,  $<$ , or  $=$  respectively. If there is not enough information to decide, write None in the blank space.

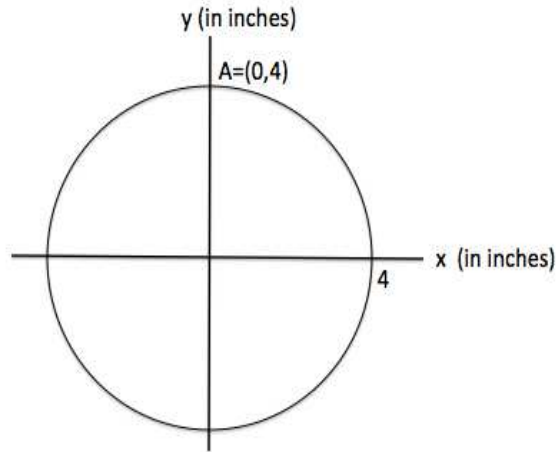
*Solution:*

$$\sin \theta > \sin(180^\circ + \theta) \quad \sin \theta = \sin(180^\circ - \theta).$$

$$\sin \theta = \sin(720^\circ + \theta) \quad \cos \theta > \sin \theta.$$

$$\cos(-\theta) = \cos \theta.$$

- c. [5 points] A beetle starts at the point  $A = (0, 4)$  on a circle with radius of 4 inches centered at the origin. The beetle walks ten seconds at a constant speed of 0.5 inches per second around the circle in the *clockwise* direction. Find the **exact** coordinates of the final location of the beetle on the circle. Show all your work.



*Solution:* The beetle walks a total distance on the circle (arc length) of 5 inches. Hence the angle between the line connecting the the point with the origin and the positive  $y$ -axis is  $\frac{5}{4}$  radians (using the arc length formula  $s = r\theta$  with  $s = 5$  and  $r = 4$ ). At that point, the angle between the line connecting the the point with the origin and the positive  $x$ -axis is  $\frac{\pi}{2} - \frac{5}{4}$ . Hence the **exact** coordinates of the final location of the beetle on the circle are

$$\left(4 \cos \left(\frac{\pi}{2} - \frac{5}{4}\right), 4 \sin \left(\frac{\pi}{2} - \frac{5}{4}\right)\right) \text{ or } \left(4 \sin \left(\frac{5}{4}\right), 4 \cos \left(\frac{5}{4}\right)\right)$$